Pitch angle distributions of solar wind's suprathermal electrons: modeling and estimation of the turbulent scattering mean-free path

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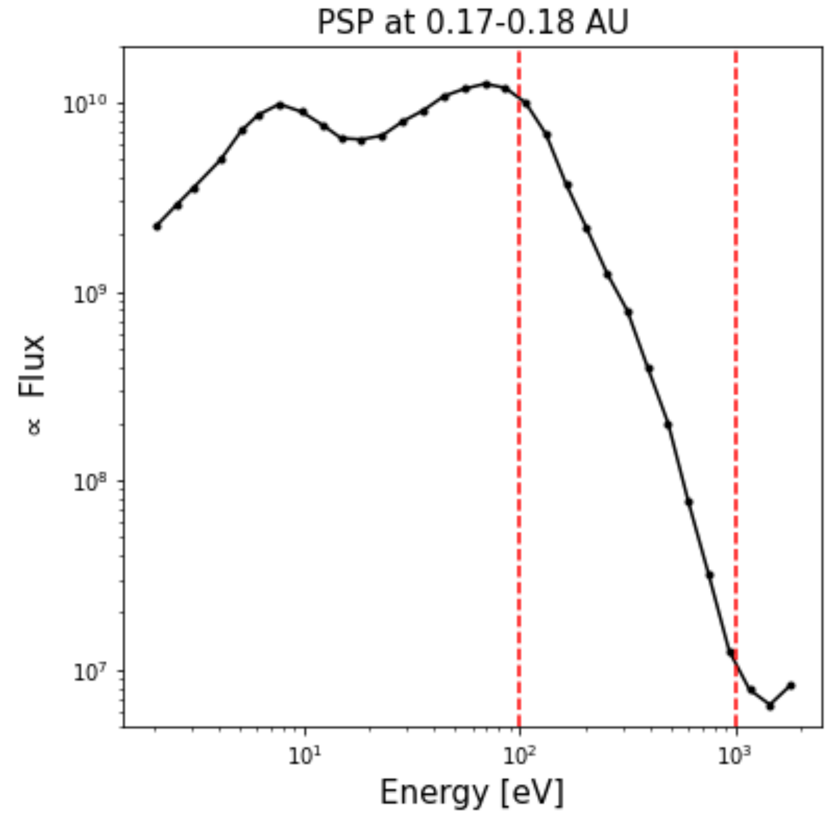
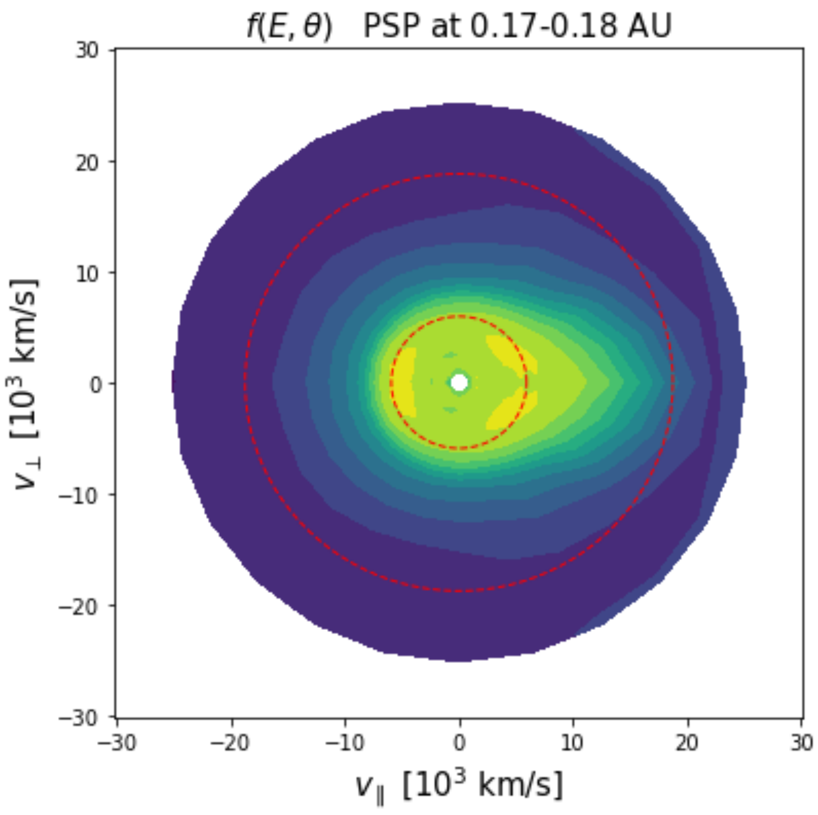
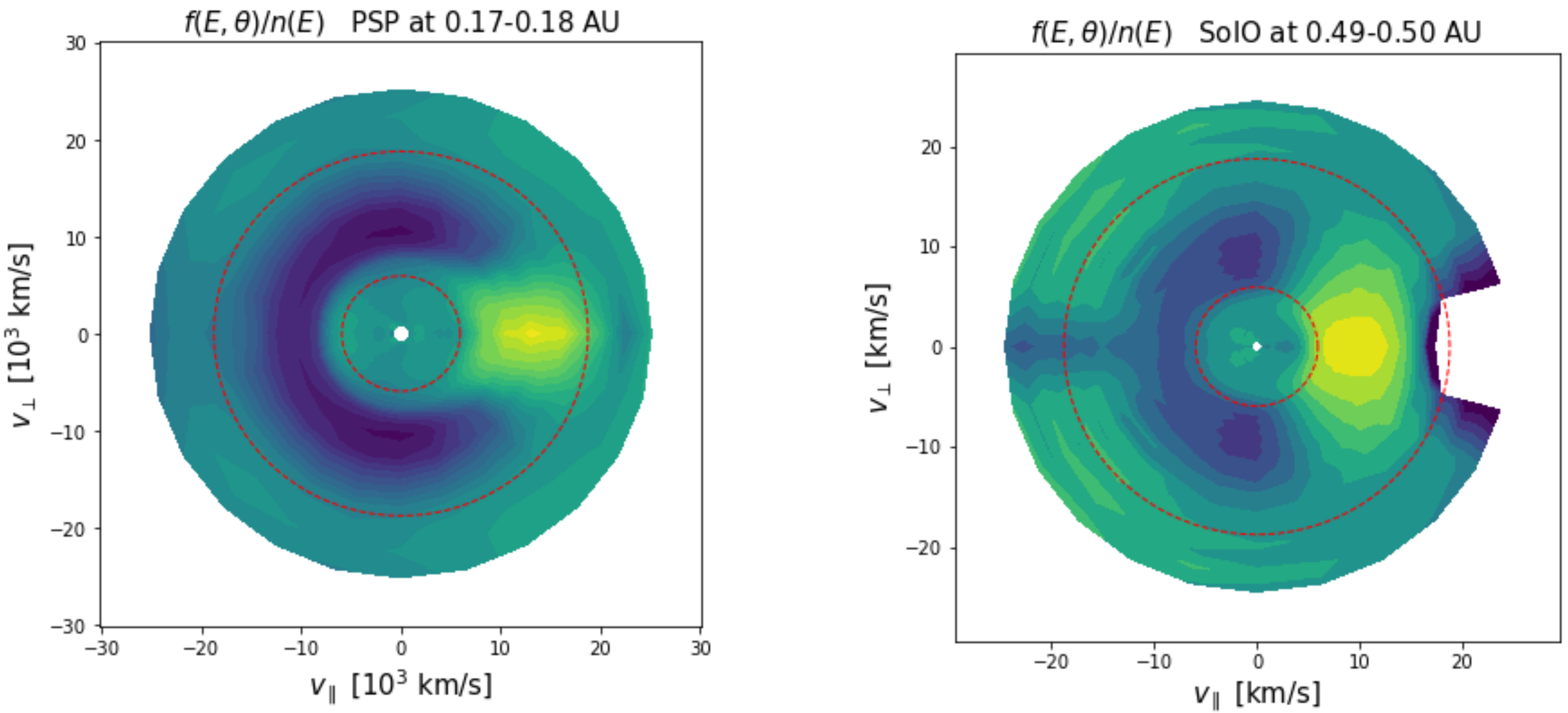
Electron velocity distribution functions in the solar wind





- Electrons vdf measured in the solar wind typically present anisotropies, in particular in the 100eV – 1keV energy range.
- These anisotropies have to be controled by some isotropization process (mirror force would collimate electrons within ~ 1° at 1 AU)

Normalized pitch-angle distributions



In the 100 eV– 1 keV range, distributions show a typical strahl/halo pitch-angle structure. The strahl angular width shows dependence both on distance and energy.

Transport equation

We use the « focused transport » equation

$$\frac{\partial f}{\partial t} + (\mathbf{V} + \mu v \mathbf{b}) \cdot \nabla f + \left\langle \frac{dv}{dt} \right\rangle_\phi \frac{\partial f}{\partial v} + \left\langle \frac{d\mu}{dt} \right\rangle_\phi \frac{\partial f}{\partial \mu} = \nu \mathcal{L}(f).$$

Which describes the evolution of the gyrophase-averaged $f(v, \mu = \cos\theta, \boldsymbol{r})$, accounting for inertial effects due to f being measured in the solar wind frame of reference.

The relevant evolution timescales are given by the expressions:

$$\left\langle \frac{d\mu}{dt} \right\rangle_\phi = \frac{1-\mu^2}{2} \left(v \nabla \cdot \mathbf{b} + \mu \nabla \cdot \mathbf{V} - 3\mu \mathbf{bb} : \nabla \mathbf{V} - \frac{2\mathbf{b}}{v} \cdot \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) - \frac{2eE_\parallel}{mv} \right)$$

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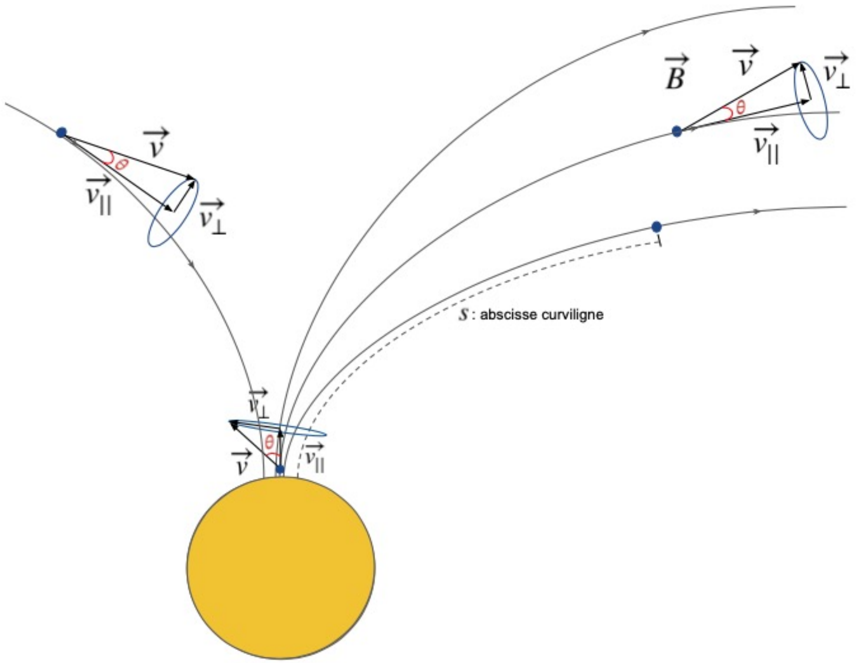
$$\frac{\nu}{\nu_{focus}} \sim \frac{v_{th}^2}{v^2} \ll 1$$

Steady state pitch angle evolution

- The energy distributions evolve on slower timescales than pitch angle distributions. So the evolution of PA distribution is to a large extent uncoupled from energy distribution.

- Keeping only the dominent terms in the transport equation, we reach the following equation for the evolution of the electrons distribution function in the $(s, \mu = \cos\theta)$ phase space



$$\mu v \frac{\partial f}{\partial s} + \frac{(1-\mu^2)v}{2L_B(s)}\frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu}\frac{(1-\mu^2)\nu}{2}\frac{\partial f}{\partial \mu}$$

Magnetic focusing by mirror force

$$\frac{1}{L_B} = \frac{1}{B}\frac{dB}{ds}$$

Isotropic pitch angle scattering by some yet unspecified process, acting on timescale $\nu = \frac{v}{\lambda}$

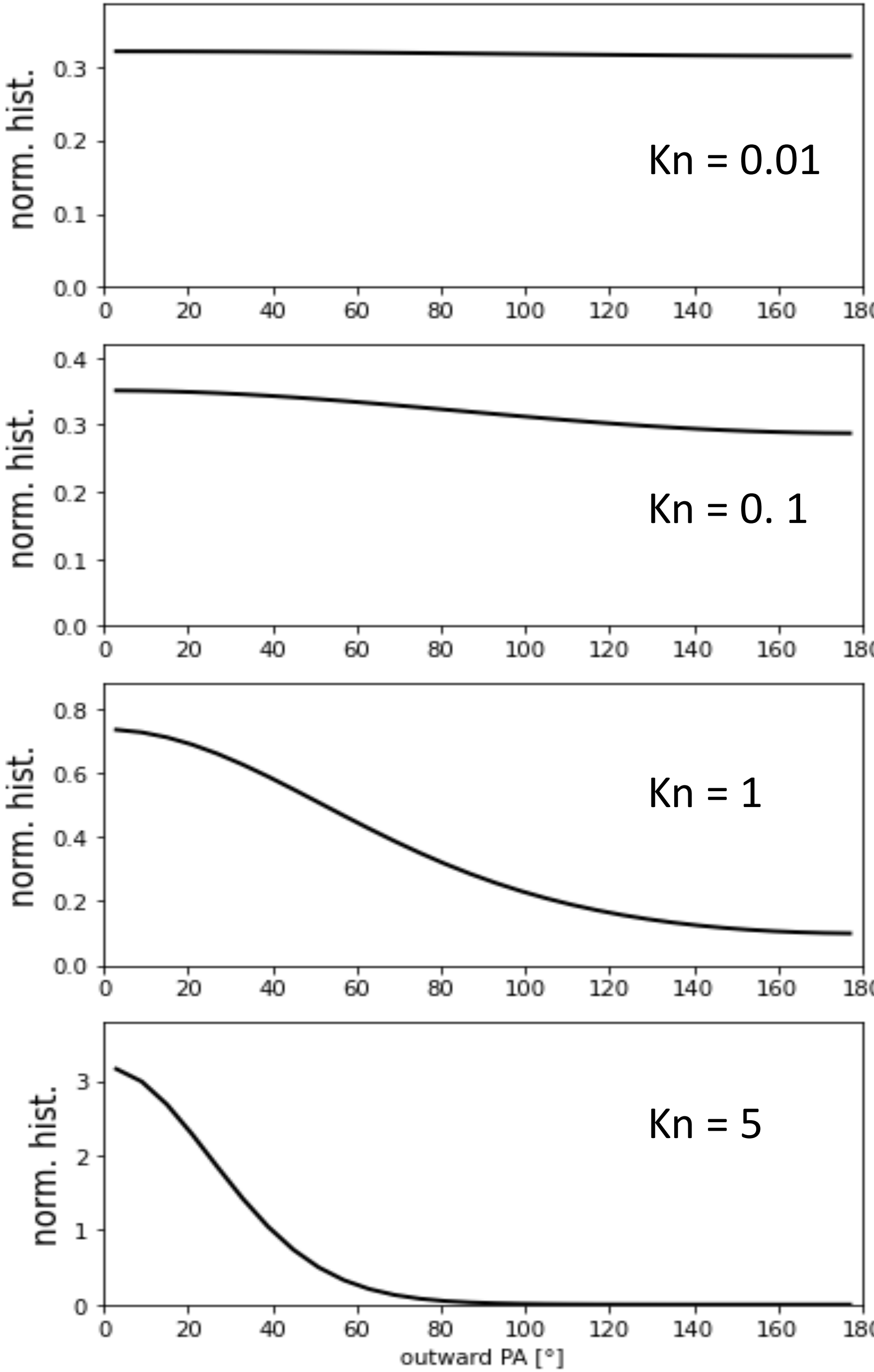
Analytical solution for a small Knudsen number

If $L_B = cste$, an analytical solution can be found for the normalized pitch angle distribution at each distance from the Sun as a function of K_n

$$f(\theta) = \frac{\mathrm{Kn}\, e^{\mathrm{Kn}\cos\theta}}{2\sinh \mathrm{Kn}}, \qquad \mathrm{Kn} = \frac{\lambda}{L_B}$$ (Knudsen number)

Sheds light on the nature of the strahl/halo pitch angle structure : produced by competition between diffusion and focusing, the balance being determined by Kn.

We expect this solution to be valid in the limit $Kn \to 0$



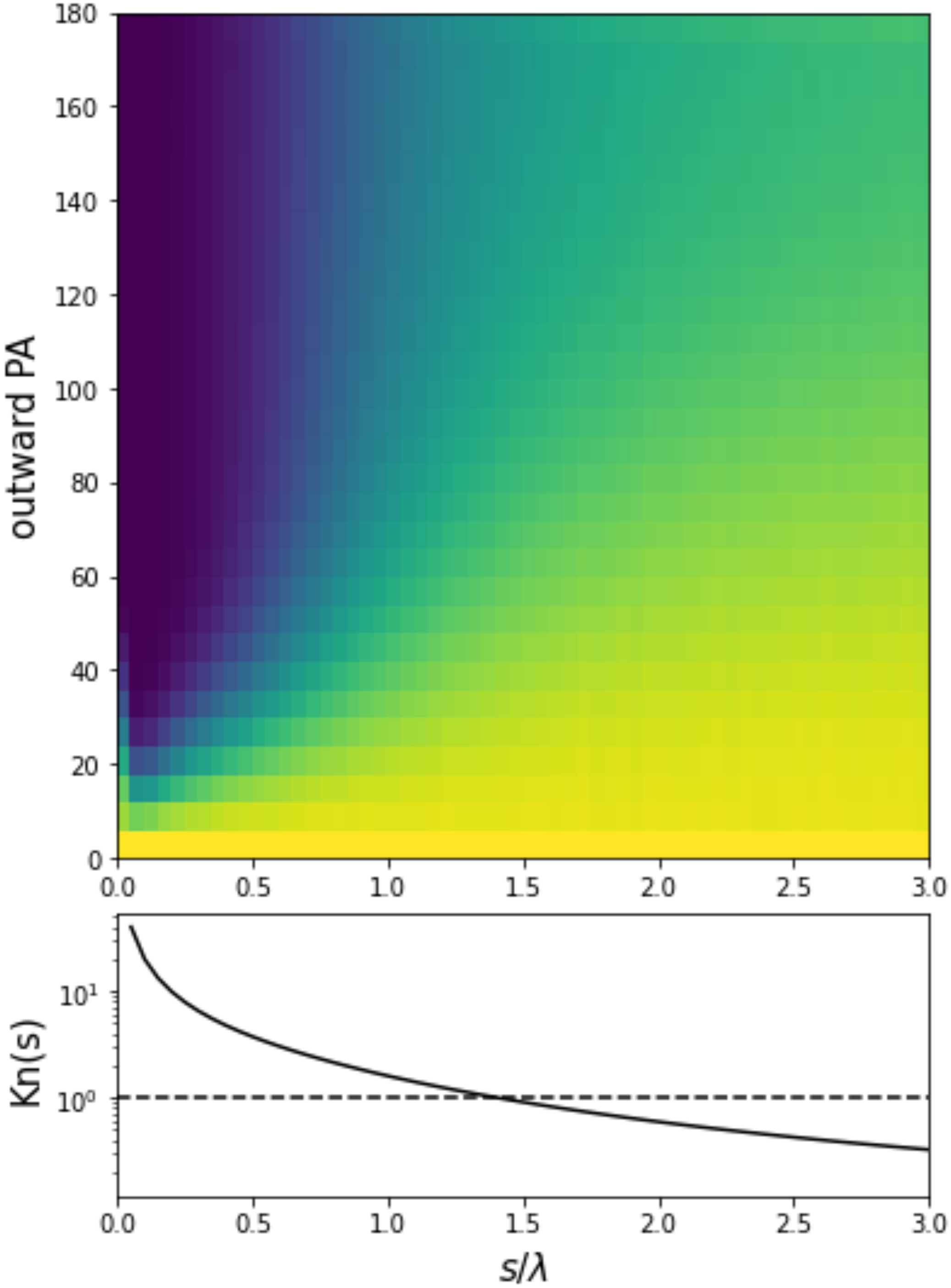
Solutions for a Parker spiral IMF profile, constant scattering mean free path

We investigate the case of a constant (independent of distance to the Sun) scattering mean-free path λ_{turb}, $\mathrm{L_B}$ is given by the Parker spiral.

The solution is obtained by numerical integration of the Fokker-Planck equation.

Boundary condition : isotropic at 0.01 AU (2 Rs) from the Sun’s center.

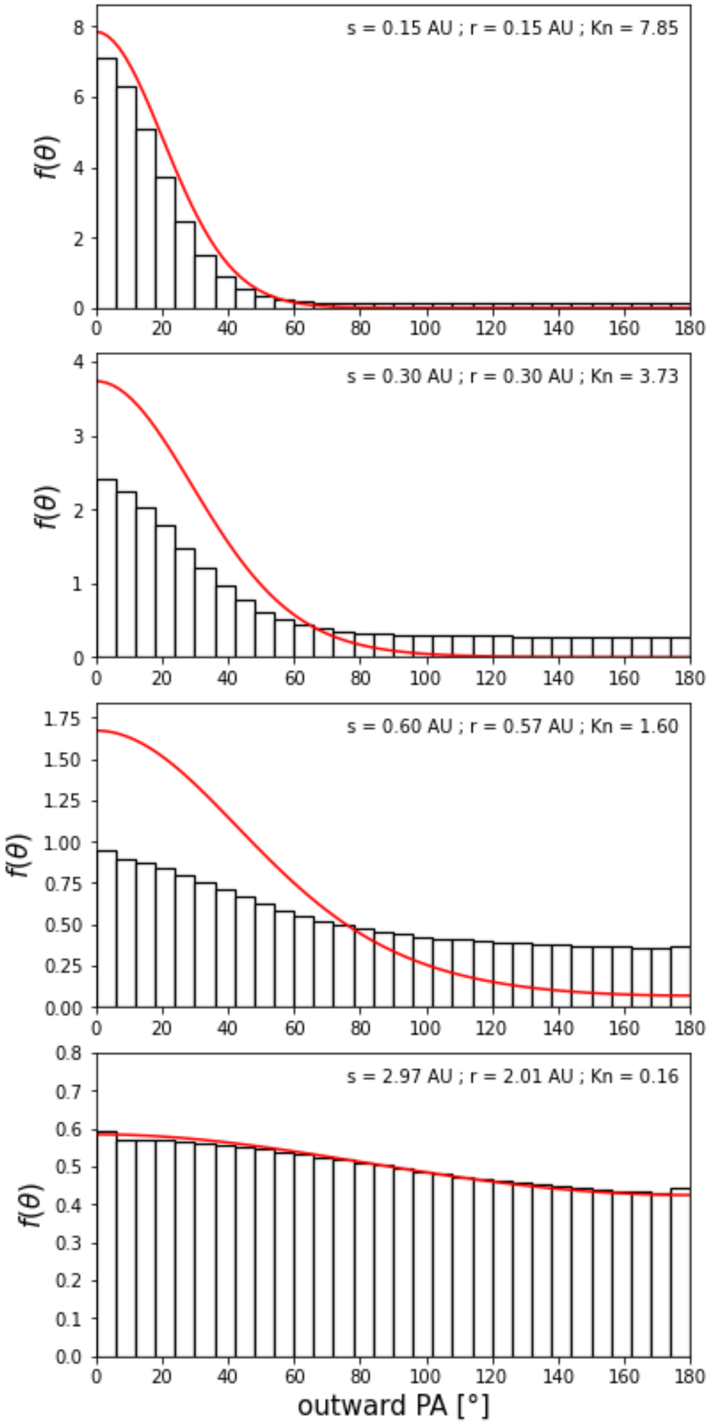
We can observed the broadeing of the « strahl » and the appearance of a « halo ».



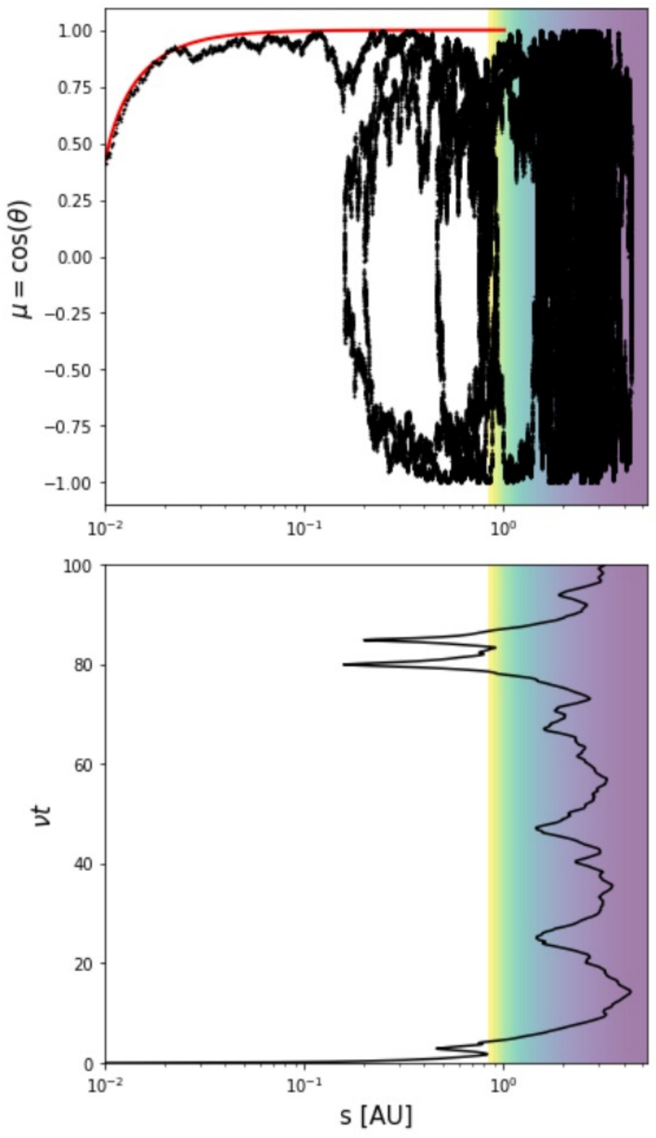
Solutions for a Parker spiral IMF profile, constant scattering mean free path

Comparision of PA distributions with the small Knudsen number limit shows the non-local behaviour of the system in large Kn regions.

Locality is recovered « far away » from the Sun, when K_n becomes small enough (typically <0.1)



What produces the distributions ? A test particle example.



- Close to the Sun, Kn is extremely high. The effect of scattering is negligible and the particle gets focused along field lines.
- The particles travels out along the field line in the region of large Kn: it is then part of the « strahl » particles.
- The particles « hits » the small Kn region at large distances (s > 1-3 AU) from the Sun. Here it gets isotropized (or « localized »): it is from now on part of the « halo » particles.
- Sometimes, the particle escapes the swamp and is injected back in the large Kn region: produces the halo component even at small distances from the Sun.

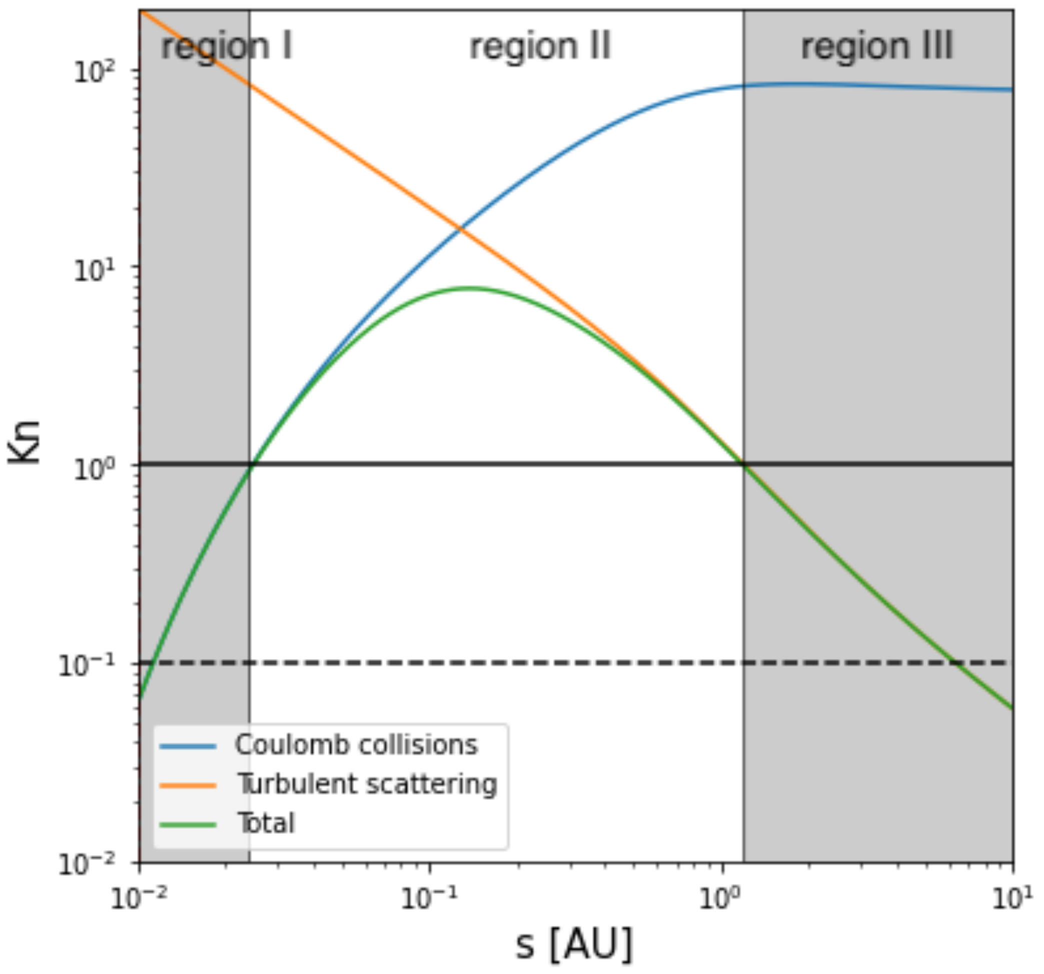
The effect of Coulomb collisions

- Close to the Sun, coulomb collisions are not negligible (because the plasma density is very high). Taking them into account using a typical (Sittler-Guhathakurta, 1999) density model, we have the following « Knudsen number » structure for the interplanetary medium

- Here the gray regions correspond to Kn<1

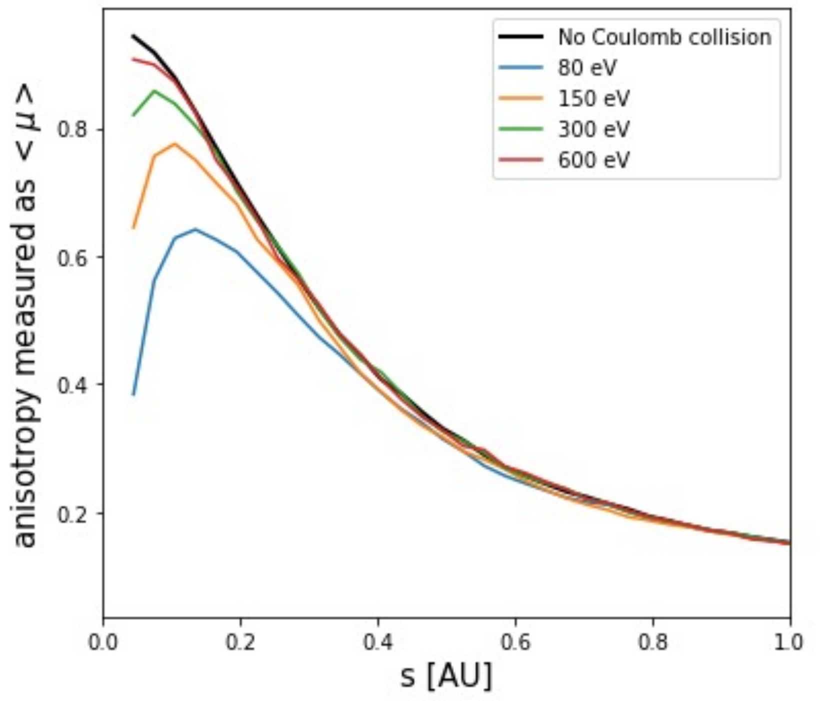
- The boundary of region I correspond to the exobase of exospheric models

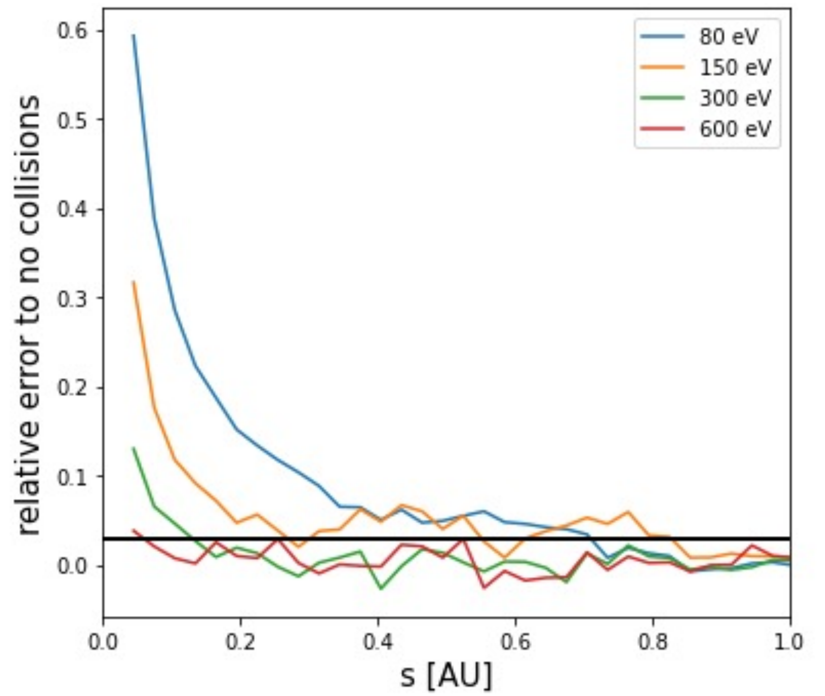
- But there is another exobase at the top... (an exoroof ?)



Where can I neglect Coulomb collisions?

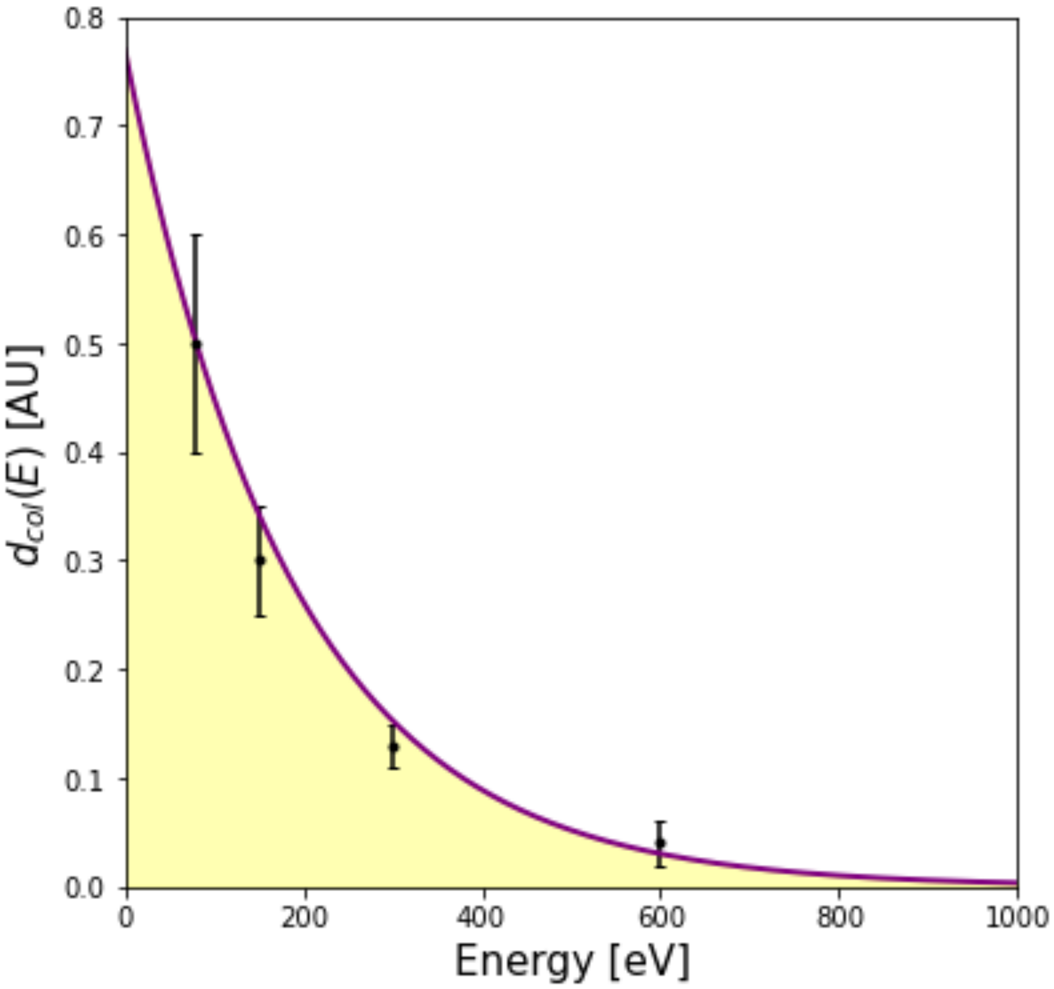
- Numerical integration of the transport equation were performed for different energies, taking coulomb collisions into account.
- The vdf's first moment (1st order anisotropy) is here plotted as a function of the curvilinear coordinate s
- After 0.6 AU roughly, distributions, even at 80 eV, are practically undistinguishable from the no-collision simulation.
- This distance diminishes when electron's energy increases (Coulomb collision cross section…)



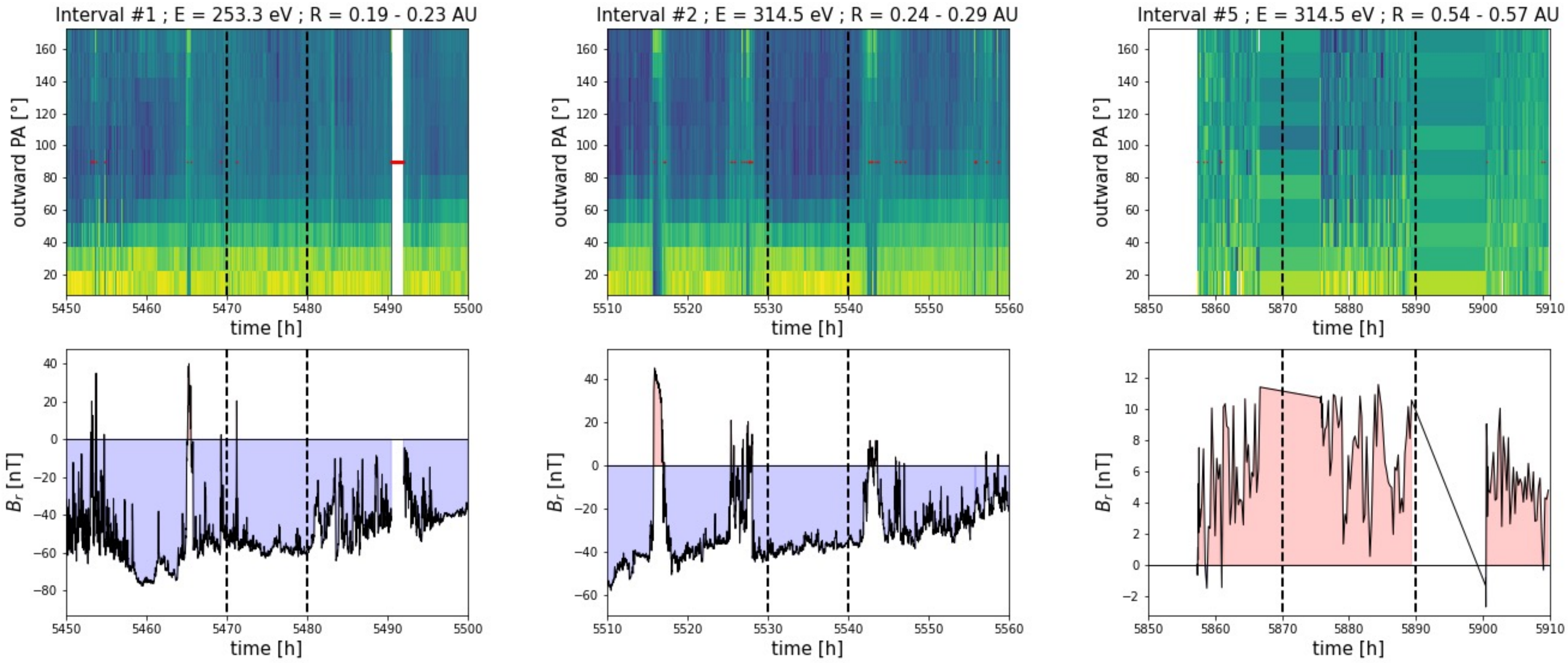


Where can I neglect Coulomb collisions?

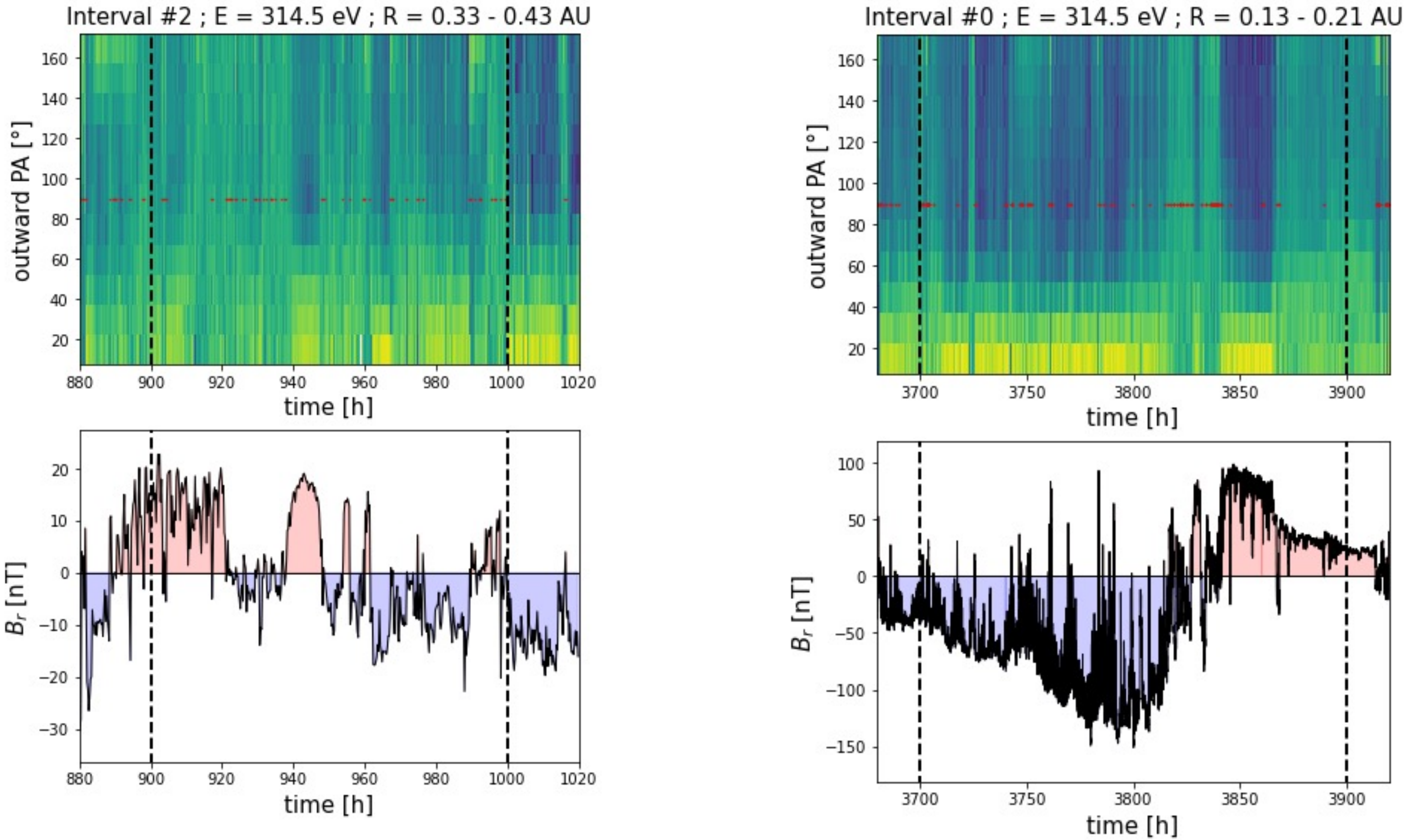
- Inside the yellow region, the distributions contain coulomb collisions effect.
- Outside the yellow region, the distributions are practically the same if I consider the combined effect of Coulomb collisions + turbulent scattering or turbulent scattering only.
- In the following we will fit the data using results of numerical integration with no coulomb collisions.



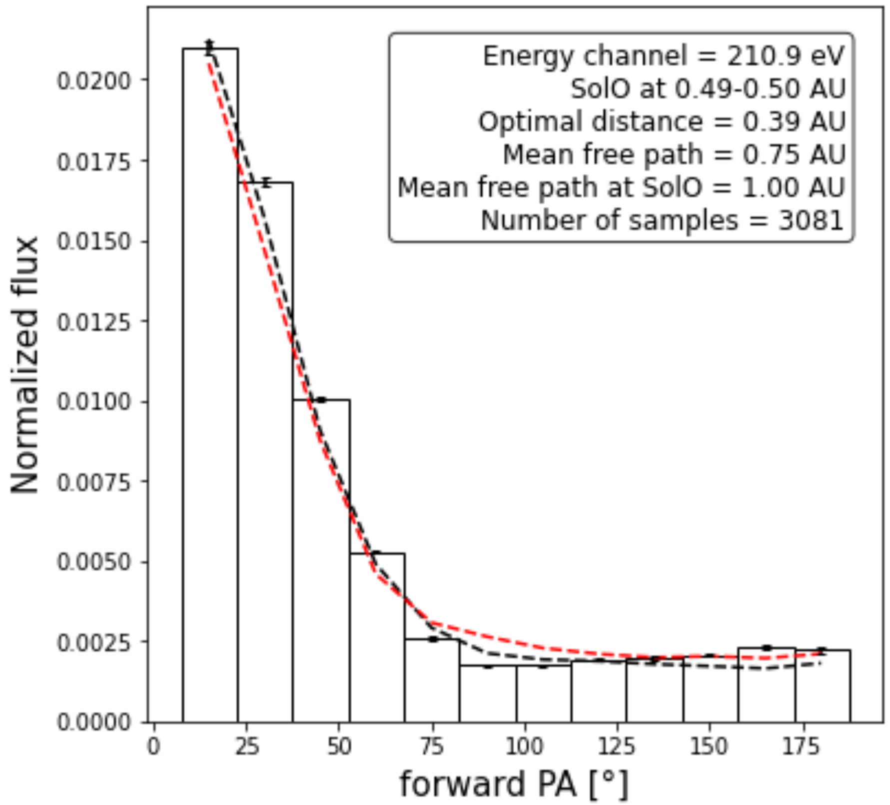
PSP SWEAP interval examples



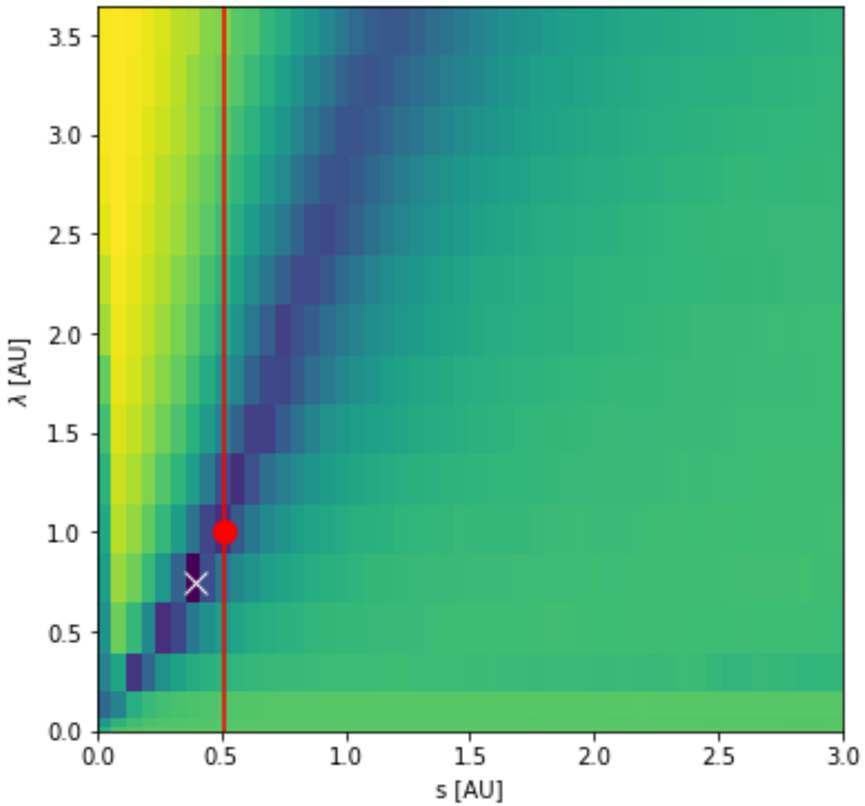
PSP SWEAP “bad/complex” interval examples



The fitting procedure (a SolO EAS example)



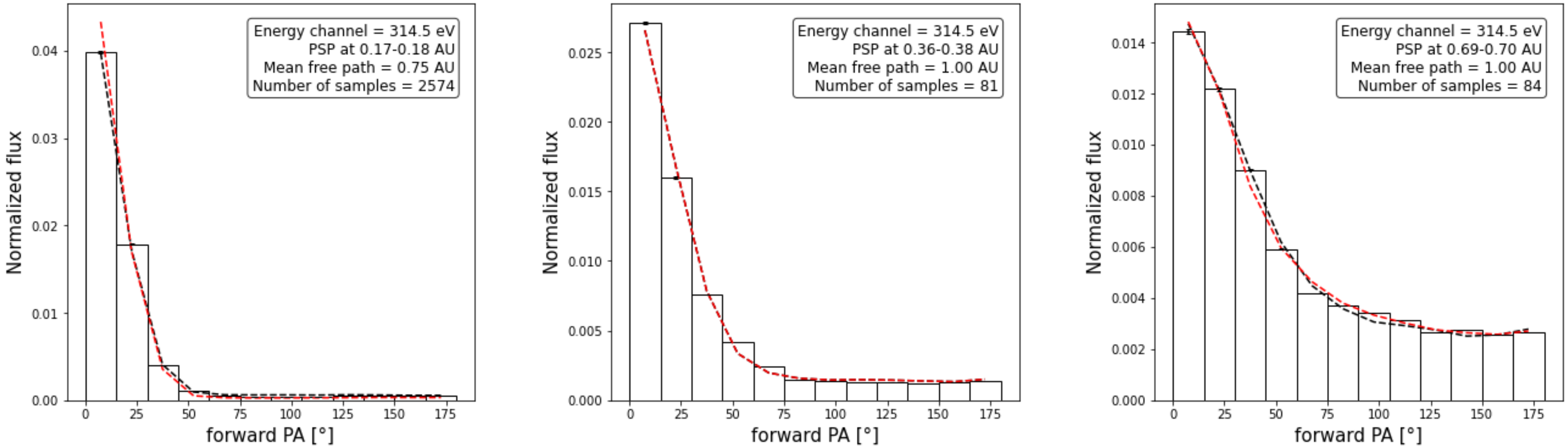
$R^2(s, \lambda)$



20 x 50 PA profiles are obtained from numerical integration of the FPE
Each of these is compared to SolO or PSP data at each energy and distance

The value of λ minimising the R^2 at the s/c position is retained.

Diffusion profiles at different distances from the Sun (PSP examples)

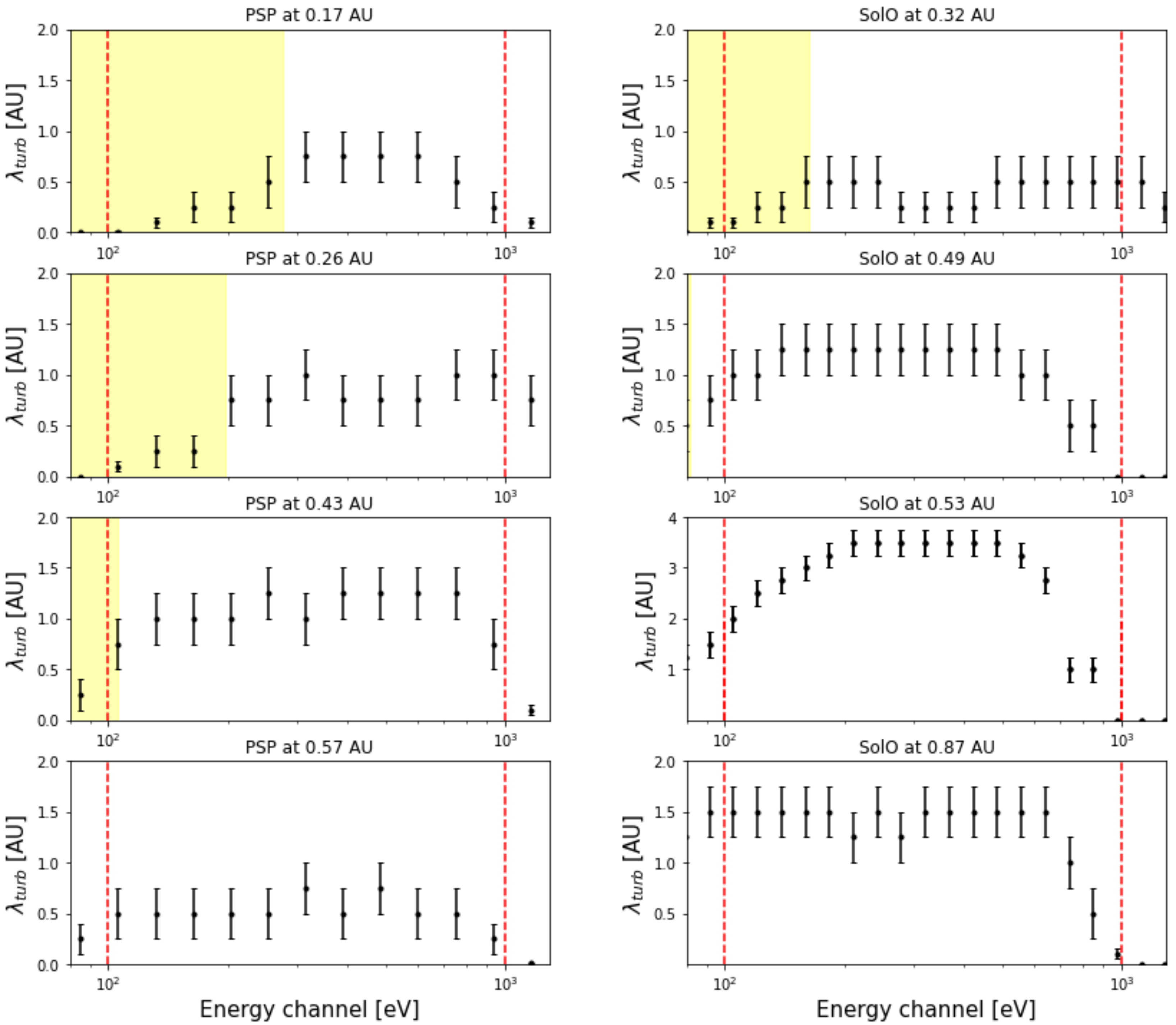


Solutions to the transport equation (with $L_B(s)$ calculated from a Parker spiral with $\psi(1AU) = 45°$) were fitted to the data, varying the mean free path λ as the only free parameter.

The results show an overall extremely good agreement: the PA distributions observed really support the scenario developped in the previous part (rather constant λ_{turb})

Mean free path as a function of the energy

- The mean free path $\lambda(E) \sim 1\ AU$ is not a strong function of the energy or distance.
- Variability from an event to another (flux tube parameters…)
- Some intervals (1/17) show « diffusion-free » profiles (with values of $\lambda \sim 3.5\ AU$ or more).



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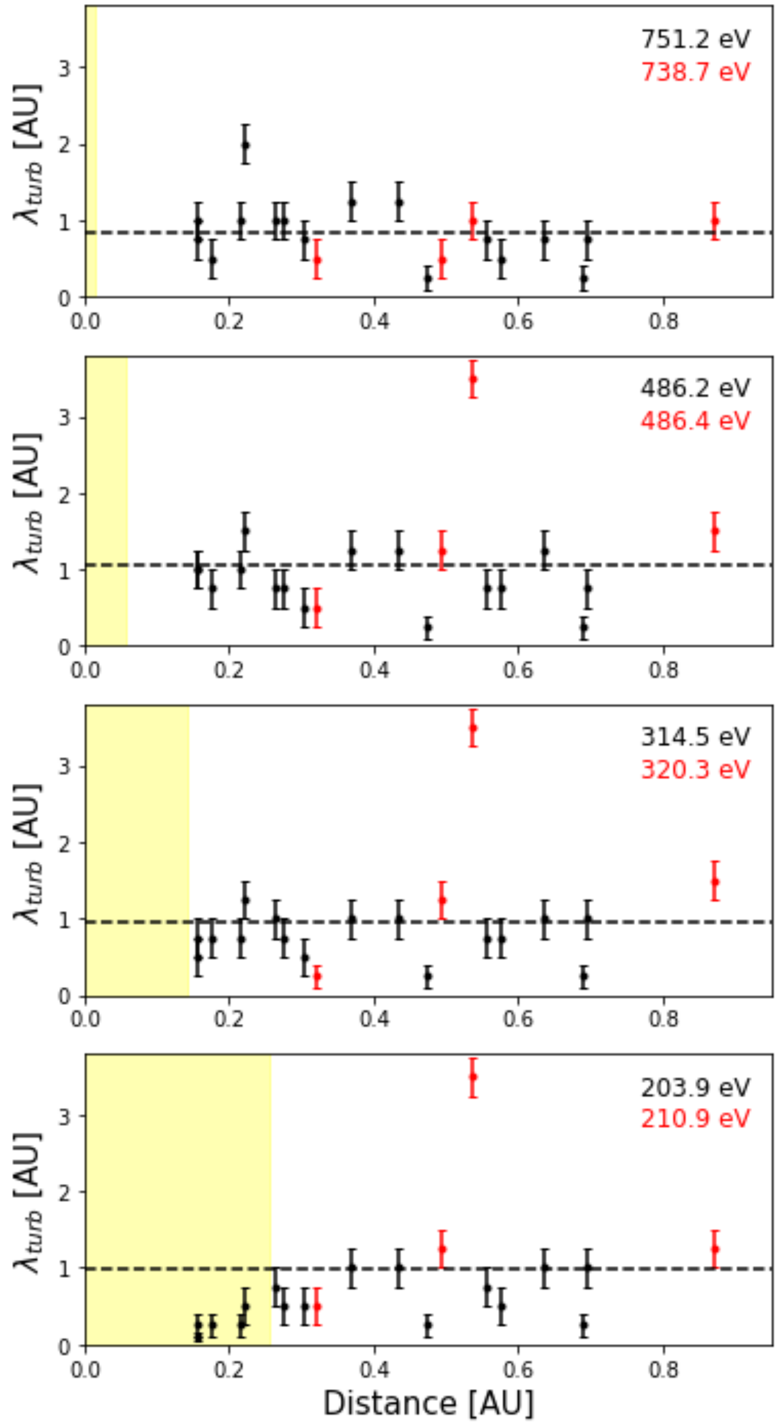

Conclusions/Summary

- The observed suprathermals PA distributions during quiet periods, and their evolution with distance to the Sun, are very accurately reproduced by a transport model with a single free parameter λ_{turb}.

- This strongly supports the existence of a turbulent scattering mechanism acting with a rather constant mean free path even at large distances from the Sun: electrons, even at high energy, evolve in a « viscous » medium.

- The halo production is given a clear explanation: halo electrons observed in large Kn regions are not locally produced, but are non-local particles having explored large portion of the field line.

- The turbulent scattering mean-free path is derived from observations as well as its energy dependence Its variability at a given distance deserves a careful parametric study (dependence on plasma beta, magnetic fluctuations, plasma density…)

Some implications and questions

- The heat flux (partly carried by suprathermals) is controlled by an isotropization process which, in most of the interplanetary medium, is not coulomb collisions.

- The existence of run-away processes in the solar wind is questionnable: the mean-free path never really increase with energy.

- What is the nature of isotropization process?
  - Whistler waves ?
  - Interaction with turbulent magnetic fluctuations ?
  - Something else ?

- The observed mean-free paths are comparable to the ones observed for SEPs events of low energies. Which are usually thought to be scattered by the interplanetary magnetic field turbulence… The constancy of λ_{turb} with distance also seems to indicate a role of the magnetic turbulence ($dB/B \sim cste$ in the interplanetary medium)