Low Frequency Receiver – LFR

status

- In-flight update of the LFR FSW since 14/03/2023 => version 3.3.0.16
- Ground segment software

Thomas Chust, Rodrigue Piberne, and the LFR team

Solar Orbiter's RPW Low Frequency Receiver (LFR)

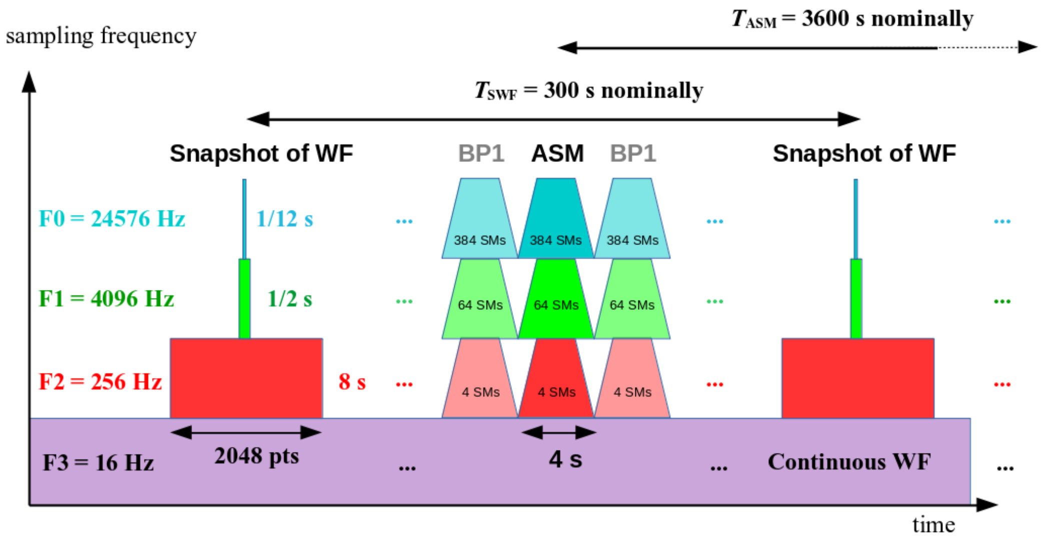
LFR nominal inputs : **3 B + 2 E + 1 V (≤ 10 kHz)**,

The LFR signal processing, based on a FPGA, provides routinely,

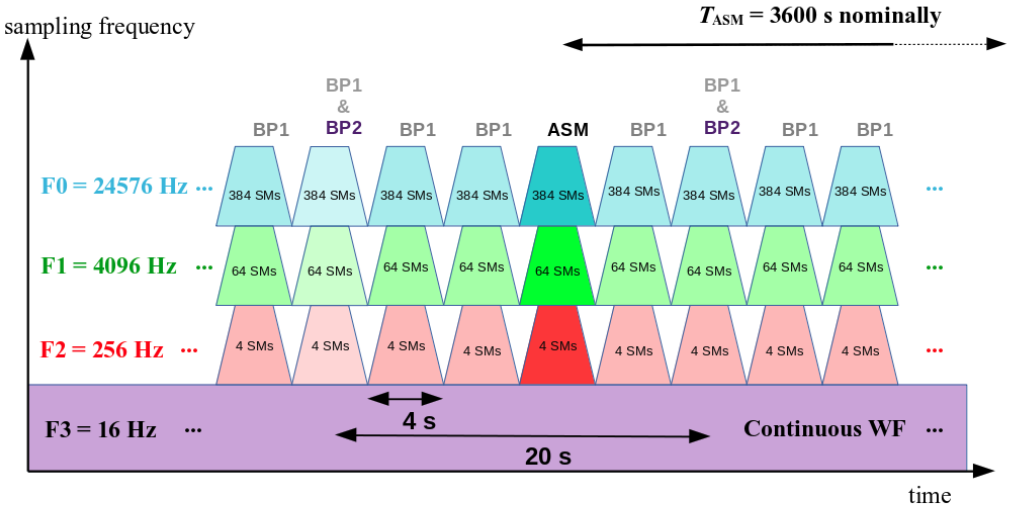
- waveforms (SWF & CWF)
- spectral matrices (ASM & BP2)
- basic wave parameters (BP1)

at different time and frequency resolutions:

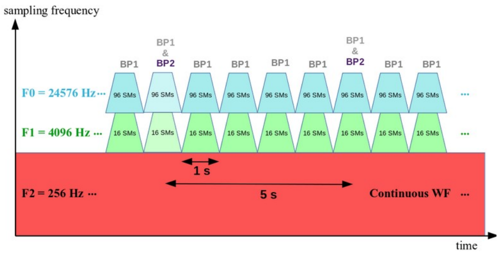
Waveforms (SWF, CWF) & Averaged spectral matrices (ASM)



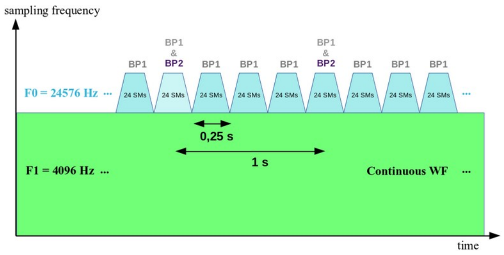
Spectral matrices (ASM, BP2) & Basic wave parameters (BP1)



Waveforms (CWF) & Spectral products (BP1, BP2)



Waveforms (CWF) & Spectral products (BP1, BP2)



LFR set of Basic Parameters before FSW update 3.3.0.16

"Instantaneous" 5 x 5 *B-E* spectral matrix (256-point FFT)

$$\mathbf{SM}\left(\omega_j^{(m)}\right)=\begin{bmatrix} B_1B_1^* & B_1B_2^* & B_1B_3^* & B_1E_1^* & B_1E_2^* \\ cc & B_2B_2^* & B_2B_3^* & B_2E_1^* & B_2E_2^* \\ cc & cc & B_3B_3^* & B_3E_1^* & B_3E_2^* \\ cc & cc & cc & E_1E_1^* & E_1E_2^* \\ cc & cc & cc & cc & E_2E_2^* \end{bmatrix}$$

m = 0, 1, 2
for F0, F1, F2

Time Averaged Spectral Matrix (ASM)

$$\mathbf{ASM}\left(\omega_j^{(m)}\right)=\frac{1}{N_{SM}^{(m)}}\sum_{k=1}^{N_{SM}^{(m)}}\mathbf{SM}_k\left(\omega_j^{(m)}\right)=\langle\mathbf{SM}\rangle_{time}$$

Frequency average ...

$$\mathbf{S}\left(\omega_j^{(m)}\right)=\langle\mathbf{ASM}\rangle_{frequency}$$

... before computations of the BPs (i.e. wave parameters)

Mono-**k** assumption: (Means, JGR, 1972) (Samson & Olson, GJRA, 1980)

$$\mathbf{n}\times\mathbf{E}=\frac{\omega}{k}\mathbf{B}$$

$$\frac{S_{ij}}{\sqrt{S_{ii}S_{jj}}}$$

BP1 PB:	Power spectrum of the magnetic field (***B***)
BP1 PE:	Power spectrum of the electric field (***E***) **=> kcoef**
BP1 NVEC:	Wave normal vector (from ***B***)
BP1 ELLIP:	Wave ellipticity estimator (from ***B***)
BP1 DOP:	Wave planarity estimator (from ***B***)
BP1 SX:	X_{SRF} (radial)-component of the Poynting vector **=> kcoef**
BP1 VPHI:	Phase velocity estimator **=> kcoef (patch needed)**
BP2 AUTO:	Autocorrelations
BP2 CROSS:	Normalized cross correlations

LFR FSW Update 3.3.0.16

First goal :
- implement the calculation of a phase velocity estimator VPHI (=> BP1)

Positive side effects :
- few optimisations and code refactoring
- few bug fixed
- in particular, the time averaging of the instaneous spectral matrices @ F1 & F0
=> ASM, BP2 and BP1 (**full time coverage now**, 1/8 before ...)

Implementation :
- another kcoeff approach (before: direct use of kcoeffs for computation of the BP1s)
- global approach of the calibration and transformation into SRF of the ASM
- **KCOEFFs are now used to upload calibration matrices (including change of reference frame) to be applied, after interpolation, on the ASM**

LFR set of Basic Parameters after FSW update 3.3.0.16

"Instantaneous" 5 x 5 spectral matrix (256-point FFT)

$$\mathbf{SM}\left(\omega_j^{(m)}\right) = \begin{bmatrix} B_1B_1^* & B_1B_2^* & B_1B_3^* & B_1E_1^* & B_1E_2^* \\ cc & B_2B_2^* & B_2B_3^* & B_2E_1^* & B_2E_2^* \\ cc & cc & B_3B_3^* & B_3E_1^* & B_3E_2^* \\ cc & cc & cc & E_1E_1^* & E_1E_2^* \\ cc & cc & cc & cc & E_2E_2^* \end{bmatrix}$$

m = 0, 1, 2
for F0, F1, F2

Time Averaged Spectral Matrix (ASM)

$$\mathbf{ASM}\left(\omega_j^{(m)}\right) = \frac{1}{N_{SM}^{(m)}} \sum_{k=1}^{N_{SM}^{(m)}} \mathbf{SM}_k\left(\omega_j^{(m)}\right) = \langle \mathbf{SM} \rangle_{time}$$

Calibration, SRF, and *frequency* average ...

$$\mathbf{S}\left(\omega_j^{(m)}\right) = \langle \mathbf{ASM\_calibrated\_SRF} \rangle_{frequency}$$

... before computations of the BPs (i.e. wave parameters)

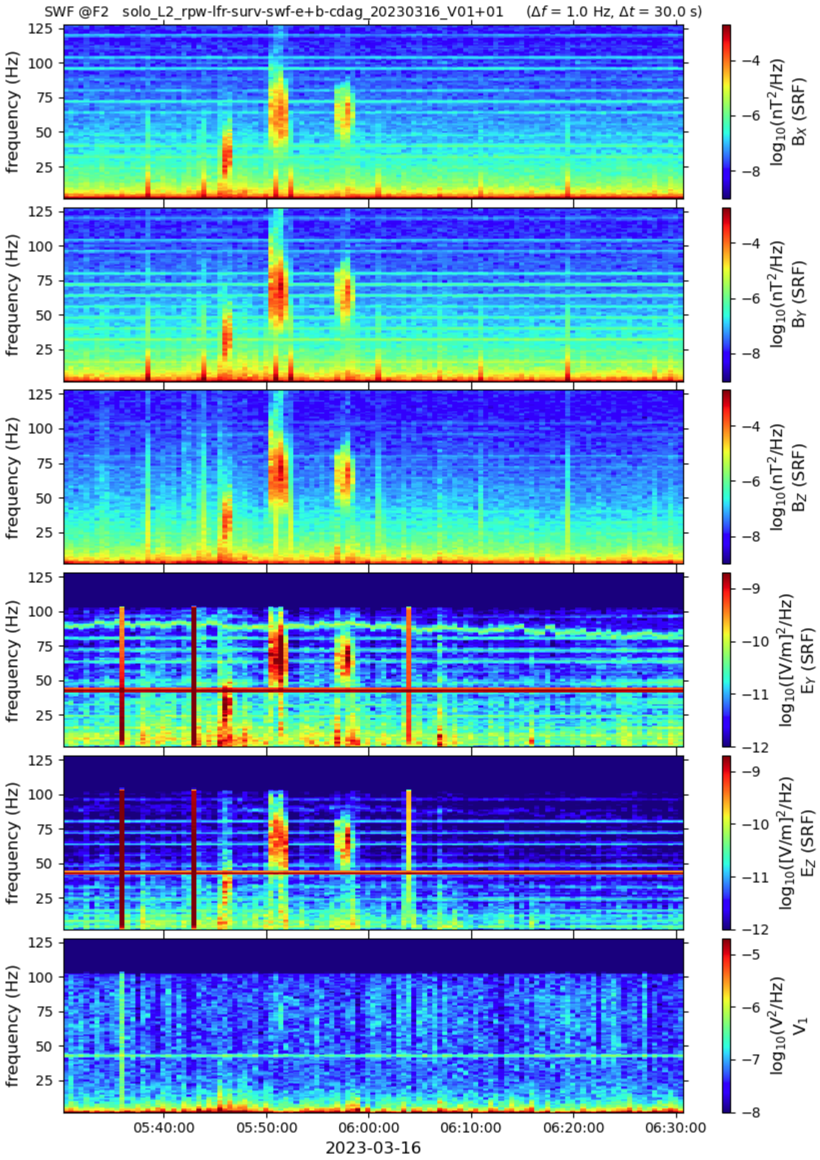
Mono-**k** assumption: (Means, JGR, 1972) — BP1 NVEC, BP1 ELLIP

(Samson & Olson, GJRA, 1980) — BP1 DOP

$$\mathbf{n} \times \mathbf{E} = \frac{\omega}{k}\mathbf{B} \longrightarrow \text{BP1 VPHI}$$

$$\frac{S_{ij}}{\sqrt{S_{ii}S_{jj}}} \longrightarrow \text{BP2 CROSS}$$

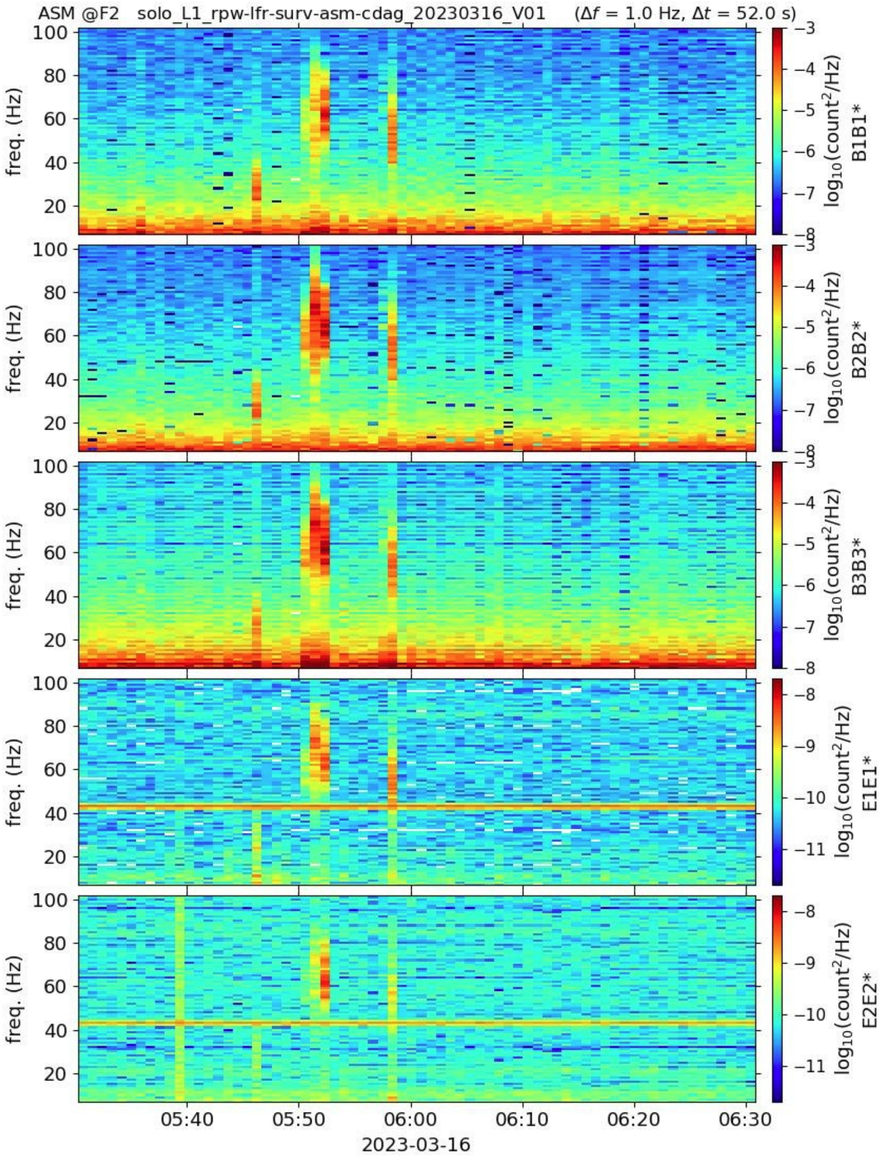
BP1 PB:	Power spectrum of the magnetic field (***B***)
BP1 PE:	Power spectrum of the electric field (***E***)
BP1 NVEC:	Wave normal vector (from ***B***)
BP1 ELLIP:	Wave ellipticity estimator (from ***B***)
BP1 DOP:	Wave planarity estimator (from ***B***)
BP1 SX:	X_{SRF} (radial)-component of the Poynting vector
BP1 VPHI:	Phase velocity estimator
BP2 AUTO:	Autocorrelations
BP2 CROSS:	Normalized cross correlations

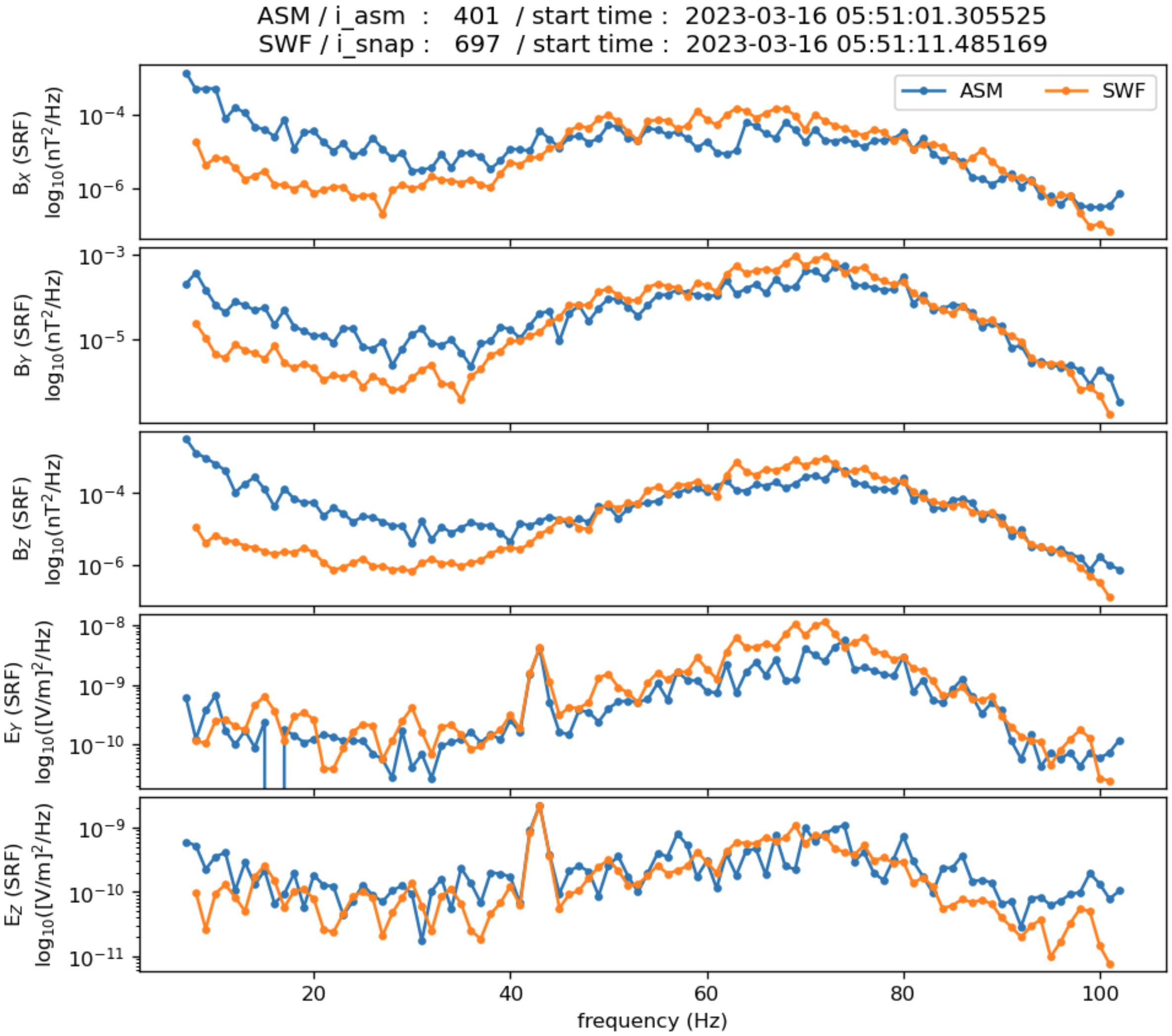


SWF

versus

ASM





ASM / i_asm : 401 / start time : 2023-03-16 05:51:01.305525
SWF / i_snap : 697 / start time : 2023-03-16 05:51:11.485169
ASM
SWF
B_X (SRF) log10(nT^2/Hz)
B_Y (SRF) log10(nT^2/Hz)
B_Z (SRF) log10(nT^2/Hz)
E_Y (SRF) log10([V/m]^2/Hz)
E_Z (SRF) log10([V/m]^2/Hz)
frequency (Hz)

LFR ground segment software

Status

Current status

- L1 to L1R pipeline produces : - CWF in SBM1, SBM2 and SURV mode.
  - SWF in SURV mode.
- L1 to L2 pipeline produces : - BP1 in SBM1, SBM2 and SURV mode.
  - BP2 in SBM1, SBM2 and SURV mode.
  - ASM in SURV mode.
- Summary plots for BP1, BP2 and ASM (L1 and L2) and CWF and SWF (L1).

Recent updates

- Management of the new calibration induced by the update of the onboard calibration (new k-coefficients). This concerns only data after the 14/03/2023.
- Deliver summary plots for BP1, BP2 and ASM in SRF frame.

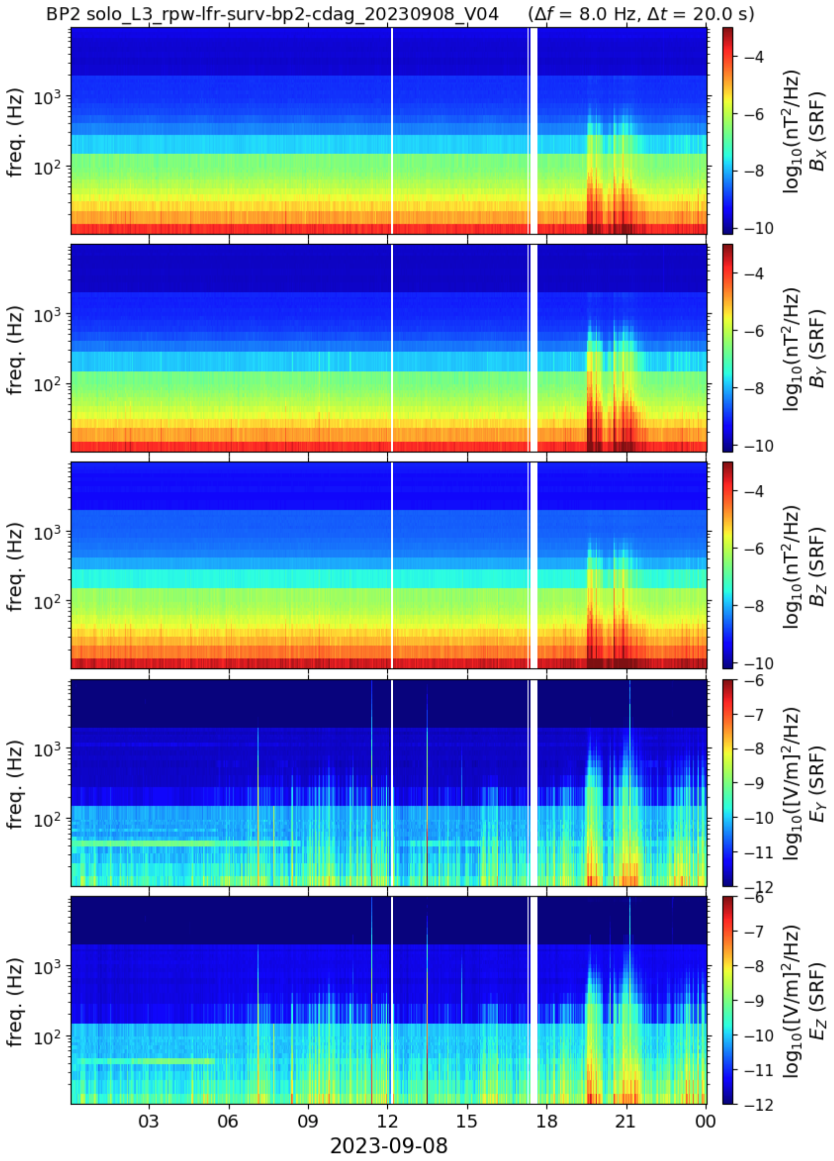
Upcoming upgrades

- Create new L3 products for ASM, BP1 and BP2 and associated summary plots.

=> Timing problems encountered

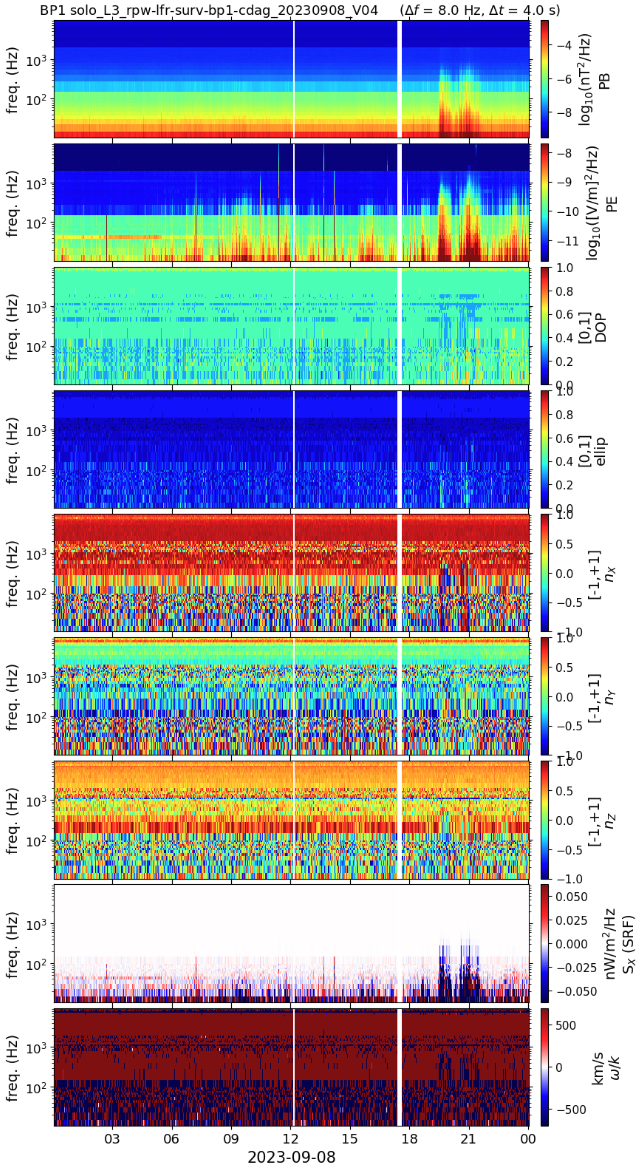
L3 LFR spectral data

(plot exemples)



BP2

BP1



LFR ground segment software
L3 timing problems

What are the LFR L3 ?

The L3 for BP1, BP2 and ASM are concatenations of the L2 according to the frequency axis:

Ex: For **L2** SURV BP1, ellipticity in normal mode is defined by:

ELLIP_N_F0, **ELLIP_N_F1**, **ELLIP_N_F2**

that depend on:

Epoch_N_F0, **Epoch_N_F1** and **Epoch_N_F2**.

For **L3** SURV BP1, ellipticity in normal mode is defined by:

ELLIP_N

that depends on:

Epoch_N.

What is the problem ?

The Epoch of the L3 is a mean of the Epochs of the L2. Meanwhile there should be always the same number of times for F0, F1 and F2 (by construction), it appears that sometimes, the Epochs of L2 data do not have the same number of times.

LFR ground segment software

L3 timing problems

When does it happen?

A test was performed over 3 months (April, May and June 2020). Results are:

	Ex: F0 | F1 | F2
3 days with 1 more F2 point	255 | 255 | 256
2 days with 1 more F0 point	256 | 255 | 255
1 day with a lack of 3 F2 points	255 | 255 | 252
1 day with a lack of 2 F2 points	255 | 255 | 253
3 days with a lack of 1 F0 point	254 | 255 | 255
4 days with a lack of 1 F2 point	255 | 255 | 254
1 day with different times for each frequency	253 | 255 | 254

15 days with problems / 91 days

LFR ground segment software

L3 timing problems

Why?

Possible causes are:
- Telecom errors?
- Decommutations errors? (Could it be checked by LESIA ? Issue on gitlab)
- Instrument errors?

What to do?

⇒Try to find the cause and solve the problem at L1 level

⇒ If not possible, put fill values at L3 level.

More complicated that it seems (need to create a new Epoch).

Conclusion

- LFR FSW update 3.3.0.16 (since 14/03/2023) is working well so far
- The switch from V12_AC to V13_AC (since 21/08/2023) would require an update of the kcoeffs in the near future (Sx and VPHI)
- The implementation of the L3 LFR spectral products is ongoing

Additional slides

LFR spectral frequencies

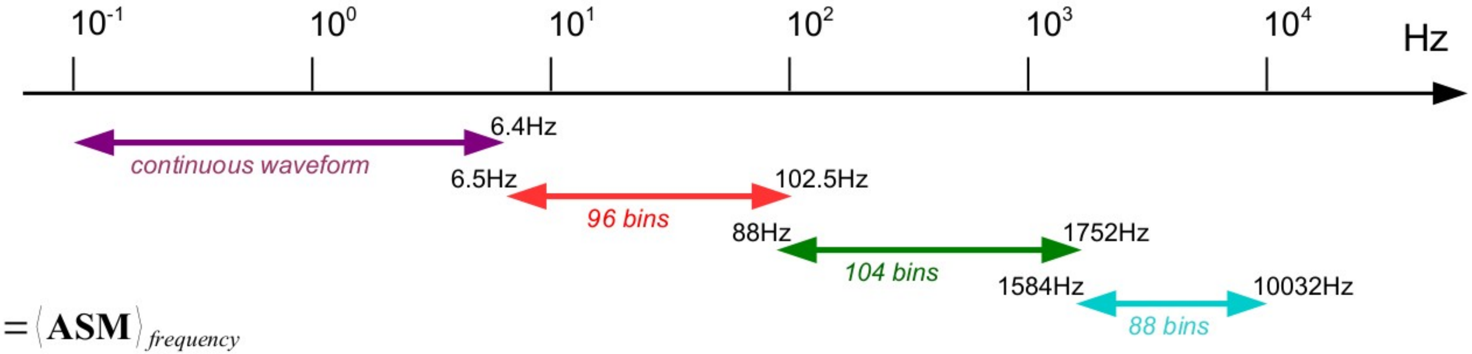
(1) Depending on the frequency channel, **selection** of 96, 104 or 88 consecutive **frequency bins** among 128 (N_{FFT} = 256) of the *time* averaged spectral matrices.

(2) Then, the ASMs are averaged over packets of N_{freq} (8 or 4) consecutive bins :

$$\Delta f^{(m)} = \frac{f_m}{N_{FFT}} \times N_{freq}$$

$$N_{freq} = 8$$

f_3 = 16 Hz	=>	waveform	[DC, 8Hz]		f_3 / 2.5 = 6.4 Hz
f_2 = 256 Hz	=>	12 frequencies	[6.5Hz, 102.5Hz]	$\Delta f^{(2)}$ = 8 Hz	f_2 / 2.5 = 102.4 Hz
f_1 = 4096 Hz	=>	13 frequencies	[88Hz, 1752Hz]	$\Delta f^{(1)}$ = 128 Hz	f_1 / 2.5 = 1638.4 Hz
f_0 = 24576 Hz	=>	11 frequencies	[1584Hz, 10032Hz]	$\Delta f^{(0)}$ = 768 Hz	f_0 / 2.5 = 9830.4 Hz



$$\mathbf{S} = \langle \mathbf{ASM} \rangle_{frequency}$$

Update of the LFR onboard k-coefficients (16 first / 32)

Has been done at the beginning of STP103 (06/07-12/07)

```
:       SOLO_CAL_RPW_BIAS_V202003101607.cdf
        SOLO_CAL_RPW-SCM_SCM-FS-MEB-PFM_V20200428000000.cdf
used:   SOLO_CAL_RCT-LFR-BIAS_V20190123171020.cdf
sed:    SOLO_CAL_RCT-LFR-SCM_V20190123171020.cdf
:       AC_DIFF_G5
        R = 0
        2020-06-18
```

PE : transformation into SRF (2 ortho comp.)
SX : same for B + E-B relative calibration

requency (Hz)	kcoeff_1 (float)	kcoeff_2 (float)	kcoeff_3 (float)	kcoeff_4 (float)	kcoeff_5 (float)	kcoeff_6 (float)	kcoeff_7 (float)	kcoeff_8 (float)	kcoeff_9 (float)	kcoeff_10 (float)	kcoeff_11 (float)	kcoeff_12 (float)	kcoeff_13 (float)	kcoeff_14 (float)	kcoeff_15 (float)	kcoeff_16 (float)
1968.00	1.000000	1.250000	1.000000	-0.000000	0.680709	0.084467	-0.075378	0.002689	-0.006633	-0.004708	-0.828601	-0.100358	-0.615400	-0.121279	-0.684965	-0.072993
2736.00	1.000000	1.250000	1.000000	-0.000000	0.683845	0.046790	-0.077284	0.006480	-0.005590	-0.004755	-0.831959	-0.057230	-0.610072	-0.080905	-0.688697	-0.009801
3504.00	1.000000	1.250000	1.000000	-0.000000	0.684479	0.023171	-0.079053	0.009339	-0.004768	-0.004780	-0.831838	-0.030256	-0.601822	-0.055823	-0.691407	0.035243
4272.00	1.000000	1.250000	1.000000	-0.000000	0.684285	-0.000440	-0.080385	0.012018	-0.003927	-0.004456	-0.830962	-0.002683	-0.592844	-0.031602	-0.694529	0.078141
5040.00	1.000000	1.250000	1.000000	-0.000000	0.683102	-0.032862	-0.080902	0.015643	-0.003207	-0.004357	-0.828408	0.035232	-0.582548	0.000017	-0.694929	0.128425
5808.00	1.000000	1.250000	1.000000	-0.000000	0.679045	-0.077452	-0.080685	0.020638	-0.002670	-0.004105	-0.823017	0.088014	-0.569249	0.042608	-0.689826	0.190891
6576.00	1.000000	1.250000	1.000000	-0.000000	0.670473	-0.129371	-0.079861	0.026565	-0.002390	-0.003748	-0.812267	0.150402	-0.551233	0.090651	-0.677161	0.259483
7344.00	1.000000	1.250000	1.000000	-0.000000	0.660435	-0.171886	-0.078807	0.031303	-0.002101	-0.003724	-0.799841	0.200498	-0.530949	0.130834	-0.663375	0.317959
8112.00	1.000000	1.250000	1.000000	-0.000000	0.660779	-0.168973	-0.079255	0.030848	-0.001279	-0.003575	-0.798949	0.196290	-0.517772	0.135450	-0.669077	0.330162
8880.00	1.000000	1.250000	1.000000	-0.000000	0.677499	-0.077787	-0.082895	0.018871	-0.000068	-0.003632	-0.816823	0.084936	-0.519847	0.073541	-0.715673	0.249207
9648.00	1.000000	1.250000	1.000000	-0.000000	0.681470	0.022690	-0.084661	0.006578	0.000491	-0.003514	-0.819349	-0.036598	-0.515658	0.008594	-0.754707	0.152344
152.00	1.000000	1.250000	1.000000	-0.000000	0.250260	0.644123	-0.026544	-0.050484	-0.003991	-0.011837	-0.308949	-0.769155	-0.238878	-0.832557	-0.266696	-0.894962
280.00	1.000000	1.250000	1.000000	-0.000000	0.435058	0.536410	-0.045370	-0.038012	-0.004918	-0.008762	-0.530177	-0.636836	-0.422869	-0.688790	-0.463289	-0.733262
408.00	1.000000	1.250000	1.000000	-0.000000	0.533893	0.436249	-0.055709	-0.029291	-0.005497	-0.007015	-0.648099	-0.517161	-0.513439	-0.555338	-0.562708	-0.585619
536.00	1.000000	1.250000	1.000000	-0.000000	0.587681	0.359136	-0.061837	-0.022570	-0.006285	-0.006126	-0.713384	-0.424653	-0.558237	-0.453699	-0.611799	-0.472014
664.00	1.000000	1.250000	1.000000	-0.000000	0.618808	0.300719	-0.065342	-0.017659	-0.007364	-0.006185	-0.751394	-0.355742	-0.581250	-0.378724	-0.638215	-0.387033
792.00	1.000000	1.250000	1.000000	-0.000000	0.638057	0.256898	-0.067508	-0.013749	-0.007346	-0.005052	-0.774980	-0.303034	-0.594074	-0.323367	-0.652844	-0.323797
920.00	1.000000	1.250000	1.000000	-0.000000	0.650217	0.222509	-0.069381	-0.010659	-0.007270	-0.004845	-0.790133	-0.262800	-0.601775	-0.280832	-0.661824	-0.274364
1048.00	1.000000	1.250000	1.000000	-0.000000	0.659008	0.194906	-0.070656	-0.007736	-0.007396	-0.004562	-0.800838	-0.229373	-0.606966	-0.246803	-0.668286	-0.234006
1176.00	1.000000	1.250000	1.000000	-0.000000	0.665408	0.169230	-0.071854	-0.006071	-0.007568	-0.004507	-0.809481	-0.199492	-0.610956	-0.217468	-0.673320	-0.198444
1304.00	1.000000	1.250000	1.000000	-0.000000	0.670761	0.146243	-0.072662	-0.003866	-0.007289	-0.004516	-0.815947	-0.172497	-0.614190	-0.191600	-0.677752	-0.167006
1432.00	1.000000	1.250000	1.000000	-0.000000	0.674564	0.125896	-0.073488	-0.001698	-0.007126	-0.004639	-0.820815	-0.148492	-0.616342	-0.169058	-0.680765	-0.139056
1560.00	1.000000	1.250000	1.000000	-0.000000	0.677124	0.111511	-0.073890	-0.000368	-0.007175	-0.004652	-0.823828	-0.131910	-0.616715	-0.152930	-0.682387	-0.117971
1688.00	1.000000	1.250000	1.000000	-0.000000	0.677909	0.106444	-0.074376	0.000250	-0.006978	-0.004553	-0.824805	-0.125884	-0.615684	-0.145642	-0.682700	-0.106756
10.50	1.000000	1.250000	1.000000	-0.000000	-0.064163	0.691371	0.005238	-0.049219	0.001012	-0.010796	0.052509	-0.788477	0.071794	-0.856683	0.075569	-0.916724
18.50	1.000000	1.250000	1.000000	-0.000000	-0.138377	0.678801	0.008808	-0.050425	0.004210	-0.011572	0.136663	-0.788797	0.162978	-0.855399	0.174421	-0.916848
26.50	1.000000	1.250000	1.000000	-0.000000	-0.118979	0.682794	0.007025	-0.049553	0.001158	-0.010515	0.115928	-0.798322	0.146259	-0.866676	0.156269	-0.931405
34.50	1.000000	1.250000	1.000000	-0.000000	-0.086301	0.687445	0.004599	-0.050711	0.001742	-0.010776	0.079182	-0.809000	0.110229	-0.880463	0.116501	-0.944506
42.50	1.000000	1.250000	1.000000	-0.000000	-0.051160	0.691451	0.002795	-0.050467	0.002297	-0.008438	0.042965	-0.813893	0.074655	-0.888435	0.078498	-0.951999
50.50	1.000000	1.250000	1.000000	-0.000000	-0.017434	0.690637	-0.003165	-0.057706	0.007160	-0.013982	0.009704	-0.816750	0.037321	-0.898358	0.038160	-0.959488
58.50	1.000000	1.250000	1.000000	-0.000000	0.008883	0.692317	-0.001998	-0.051320	0.002576	-0.010018	-0.026732	-0.817230	0.010392	-0.894188	0.008285	-0.959643
66.50	1.000000	1.250000	1.000000	-0.000000	0.033912	0.691814	-0.006235	-0.051648	0.000089	-0.011058	-0.056269	-0.820826	-0.015915	-0.893520	-0.019992	-0.958181
74.50	1.000000	1.250000	1.000000	-0.000000	0.058759	0.689708	-0.008406	-0.051508	-0.000637	-0.010220	-0.085366	-0.818306	-0.041164	-0.890868	-0.047973	-0.954232
82.50	1.000000	1.250000	1.000000	-0.000000	0.081055	0.688097	-0.011295	-0.049474	-0.000468	-0.010783	-0.111820	-0.815674	-0.063871	-0.888986	-0.072991	-0.953335
90.50	1.000000	1.250000	1.000000	-0.000000	0.103966	0.685133	-0.012776	-0.050548	-0.001181	-0.011071	-0.137918	-0.812970	-0.087822	-0.884208	-0.099221	-0.947926
98.50	1.000000	1.250000	1.000000	-0.000000	0.122452	0.682203	-0.015249	-0.049152	-0.001474	-0.010516	-0.159991	-0.808639	-0.107694	-0.880608	-0.120736	-0.945213

11 F0 + 13 F1 + 12 F2 = 36 frequency bins

Computation of the k-coefficients for PE

Power spectrum of the electric field

$$\langle E_{Y'} E^*_{Y'} + E_{Z'} E^*_{Z'} \rangle = \left\langle {}^{\mathrm{T}}\mathbf{E}_{ANT} \cdot \frac{1}{|A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}|^2} \begin{bmatrix} |A_{2Y'}|^2 + |A_{2Z'}|^2 & -A^*_{1Y'} A_{2Y'} - A^*_{1Z'} A_{2Z'} \\ -A_{1Y'} A^*_{2Y'} - A_{1Z'} A^*_{2Z'} & |A_{1Y'}|^2 + |A_{1Z'}|^2 \end{bmatrix} \cdot \mathbf{E}^*_{ANT} \right\rangle$$

$$= \frac{|A_{2Y'}|^2 + |A_{2Z'}|^2}{|A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}|^2} \left(S_{44} + \frac{|A_{1Y'}|^2 + |A_{1Z'}|^2}{|A_{2Y'}|^2 + |A_{2Z'}|^2} S_{55} - 2\,\Re\left[\frac{A^*_{1Y'} A_{2Y'} + A^*_{1Z'} A_{2Z'}}{|A_{2Y'}|^2 + |A_{2Z'}|^2} S_{45} \right] \right)$$

Calibration factor

$$\mathrm{PE} = S_{44}\, k^{pe}_{44} + S_{55}\, k^{pe}_{55} + \Re\left[S_{45}\, k^{pe}_{45} \right]$$

with

$$\begin{cases} k^{pe}_{44} = 1 \\ k^{pe}_{55} = \dfrac{|A_{1Y'}|^2 + |A_{1Z'}|^2}{|A_{2Y'}|^2 + |A_{2Z'}|^2} \\ k^{pe}_{45} = -2\, \dfrac{A^*_{1Y'} A_{2Y'} + A^*_{1Z'} A_{2Z'}}{|A_{2Y'}|^2 + |A_{2Z'}|^2} \end{cases}$$

WARNING: The TF of BIAS and LFR are implicitly embodied in the TF matrix of ANT (just a common calibration factor)

Computation of the k-coefficients for SX

X_{SRF}-component of the Poynting vector

$$\langle S_{X'}\rangle = \langle (\mathbf{E}\times\mathbf{B}^*)_{X'}\rangle = \langle E_{Y'}\, B^*_{Z'}\rangle - \langle E_{Z'}\, B^*_{Y'}\rangle$$

$$= \left\langle \frac{A_{2Z'}\,E_1 - A_{1Z'}\,E_2}{A_{1Y'}\,A_{2Z'} - A_{1Z'}\,A_{2Y'}}\,\frac{1}{C^*_{1Y}}\,\tilde{m}^*_{Z'j}\,B^*_j\right\rangle - \left\langle \frac{-A_{2Y'}\,E_1 + A_{1Y'}\,E_2}{A_{1Y'}\,A_{2Z'} - A_{1Z'}\,A_{2Y'}}\,\frac{1}{C^*_{1Y}}\,\tilde{m}^*_{Y'j}\,B^*_j\right\rangle$$

$$= \frac{\left(A_{2Y'}\,\tilde{m}^*_{Y'j} + A_{2Z'}\,\tilde{m}^*_{Z'j}\right)\langle E_1\, B^*_j\rangle - \left(A_{1Y'}\,\tilde{m}^*_{Y'j} + A_{1Z'}\,\tilde{m}^*_{Z'j}\right)\langle E_2\, B^*_j\rangle}{\left(A_{1Y'}\,A_{2Z'} - A_{1Z'}\,A_{2Y'}\right)C^*_{1Y}}$$

Calibration factor

$$= \frac{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}}{\left(A_{1Y'}\,A_{2Z'} - A_{1Z'}\,A_{2Y'}\right)C^*_{1Y}}\left[\frac{A_{2Y'}\,\tilde{m}^*_{Y'j} + A_{2Z'}\,\tilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}}\,S_{4j} - \frac{A_{1Y'}\,\tilde{m}^*_{Y'j} + A_{1Z'}\,\tilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}}\,S_{5j}\right]$$

$$\Rightarrow \quad \mathrm{SX'} = S_{41}\,k^{sx'}_{41} + S_{42}\,k^{sx'}_{42} + S_{43}\,k^{sx'}_{43} + S_{51}\,k^{sx'}_{51} + S_{52}\,k^{sx'}_{52} + S_{53}\,k^{sx'}_{53}$$

with

$$\begin{cases} k^{sx'}_{4j} = +\dfrac{A_{2Y'}\,\tilde{m}^*_{Y'j} + A_{2Z'}\,\tilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'} - A_{1Z'}A_{2Y'}}\right)\right] & j = 1, 2, 3 \\ k^{sx'}_{5j} = -\dfrac{A_{1Y'}\,\tilde{m}^*_{Y'j} + A_{1Z'}\,\tilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'} - A_{1Z'}A_{2Y'}}\right)\right] & \end{cases}$$

WARNING: As for ANT, the TF of LFR is implicitly embodied in the TF matrix of SCM (just a common calibration factor)