Hall inertial range of turbulence

Y. Narita (IWF Graz)

1) Electron-fluid motion in the perp plane

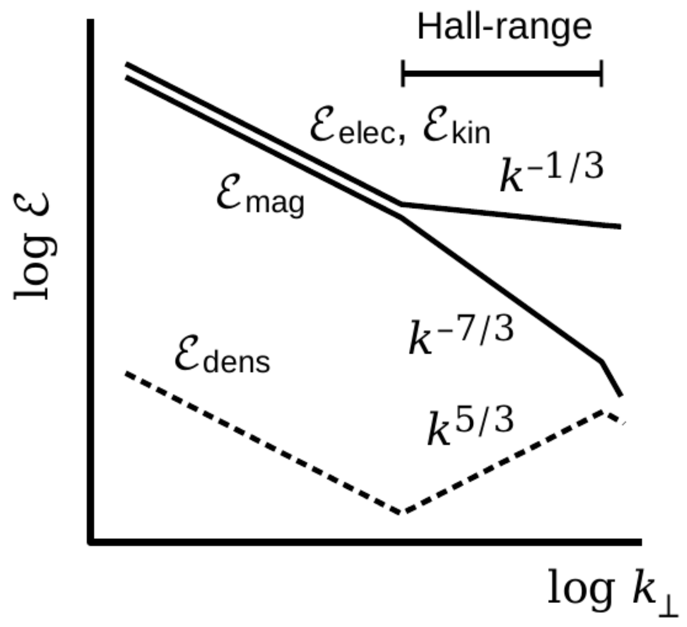
2) δB compressive sense

3) E-field electrostatic (k-perp direction)

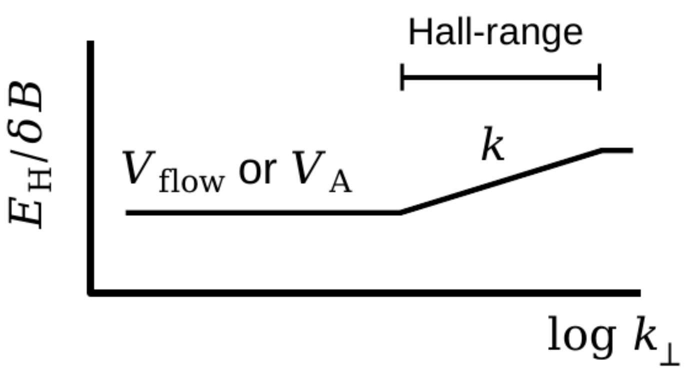
4) Application of electron-MHD scaling law to Hall electric field, yielding:

- Flattening of electric energy spectrum
- Steepening of magnetic energy spectrum
- E-B ratio no longer constant but propto k

Energy spectra



E-B ratio



Narita et al., ANGEO 2019

Minimum variance projection by Capon

Weight vector of the measured data onto the shape-vector h

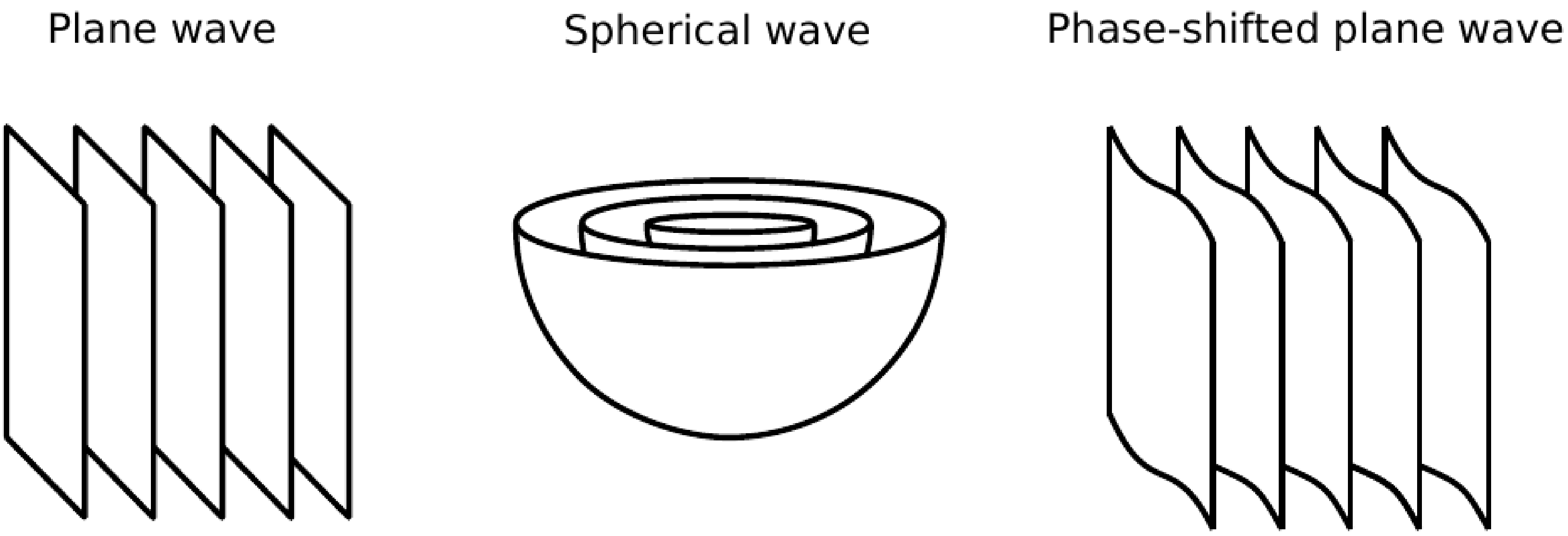
$$\vec{w} = \frac{\mathbf{R}^{-1}\vec{h}}{\vec{h}^{\dagger}\mathbf{R}^{-1}\vec{h}}$$

Projection of the covariance matrix onto the shape-vector h

$$P = \vec{w}^{\dagger}\mathbf{R}\vec{w} = \left[\vec{h}^{\dagger}\mathbf{R}^{-1}\vec{h}\right]^{-1}$$

Originally developed in the multi-point seismic data analysis by Capon (1967,1969) for a wavevector detection problem (single mode!)

Braunschweiger trio of shape vectors



Glssmeier & Motschmann (1991,1996,2001)

Constantinescu (2007)

Plaschke (2008)

The method is still for single mode.
But Capon method is applicable to multiple modes.

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Mode decomposition a la Capon

Projection of the covariance matrix onto the mode alpha and beta

$$(\mathbf{C})^{\alpha\beta} = \mathbf{C}_{\mathrm{N}} \vec{h}^{\alpha} \mathbf{N}^{-1} \mathbf{R} \mathbf{N}^{-1} \vec{h}^{\beta} \mathbf{C}_{N} \rightarrow \left[\vec{h}^{\alpha} \mathbf{R}^{-1} \vec{h}^{\beta} \right]^{-1}.$$

$$\left(\vec{\tilde{s}}^{(\mathrm{dv})} \right)_{\alpha} = (\mathbf{C}_{\mathrm{R}})_{\alpha\beta} \, (\mathbf{h})_{\beta i} \left(\mathbf{R}^{-1} \right)_{ij} \langle \vec{d}_j \rangle$$

Inputs: shape vector of each mode h
measurement covariance matrix R
measurement data d

Pro - The decomposition is essentially a fitting procedure, and does not require orthogonal basis vectors.
- Error estimate (confidence interval) has been done.

Con - Limits of the method remain unclear.
How many modes can we resolve?

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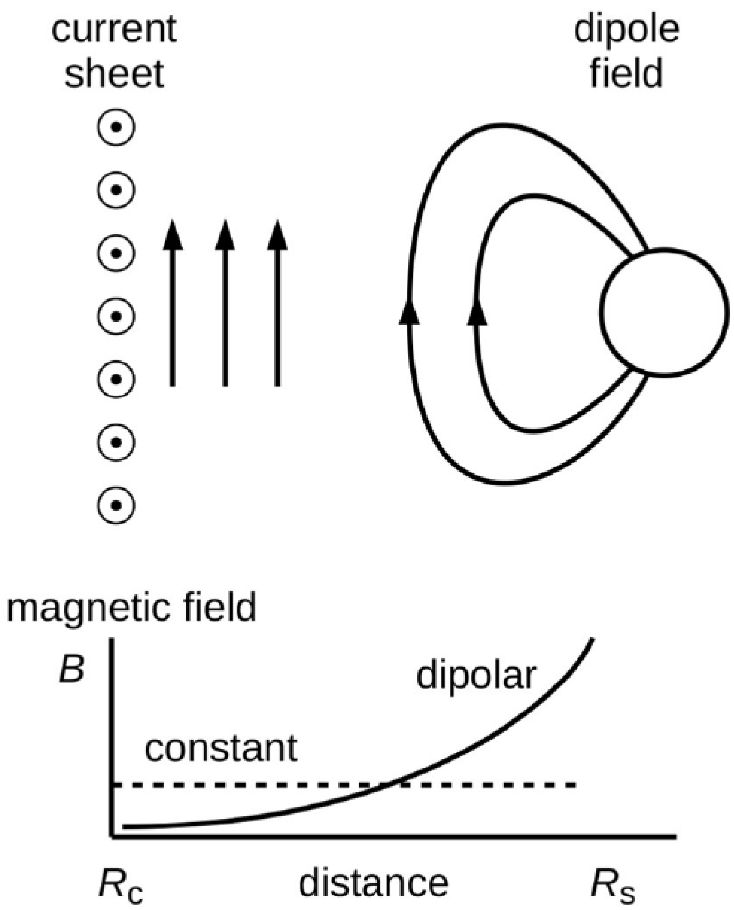
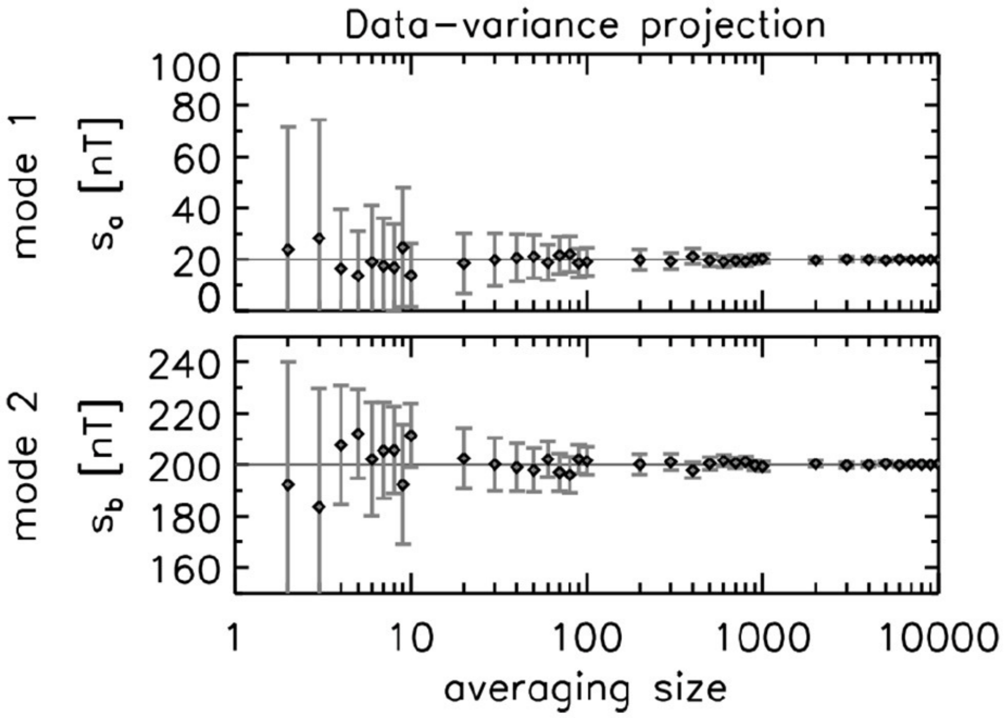
A simple test of mode decomposition

Two sensors in a static magnetic field (dipole plus external field)



Measurements over 10 times would be sufficient to decompose the field into Modes within 10%-50% accuracy.



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Dipole, quadrupole, and external field?

Result from a numerical test

Model
1. external field 20 nT
2. dipole field at surface 200 nT
3. quadrupole field at surface 80 nT

Result from two-point static Capon decomposition
1. external field 91 nT
2. dipole field -127 nT at surface
3. quadrupole field 370 nT at surface

2-point measurements do not work for 3 modes,
but BepiColombo covers radial distances
in spirit of multi-point (or multi-radial) measurements.

Question: how many multipole degrees can we resolve?