LFR status

- Alexandra Alexandrova joined LFR team with a CNES postdoctoral grant (**1st sept. 2020**)
- Ground segment software **(status)**
- Update of the k-coefficients (for usefull values of PE and SX [BP1]) effective since STP103 (06/07/2020)
  - First example with a whistler event on July 7th
- Phase shift issue between electric (BIAS) and magnetic (SCM) data ?
  - Phase velocity (VPHI) and Poynting flux (SX)

Thomas Chust, the LFR team and the RPW instrument consortium

LFR ground segment software

Since last team meeting in June 26th:

- ROC uses last version of our production pipeline [lfr-calbut v1.2.0] : WF + ASM + BP (**operational**)
- Acceleration of CALBUT production by ~factor 4 (under validation)
- Setting of the QF (ongoing)
- Calibration of ASM and BP data when BIAS is off using VHF preamplifier calibration table (under validation)
- Calibration of BP1 with k-coefficients and ANT TF (in progress)
- Calibration of SBM2 BP2 (under validation)
- Transformation of ASM and BP data into the SRF frame (to be done)

LFR current set of Basic Parameters

"Instantaneous" 5 x 5 spectral matrix
(256-point FFT)

$$\mathbf{SM}\left(\omega_j^{(m)}\right) = \begin{bmatrix} B_1B_1^* & B_1B_2^* & B_1B_3^* & B_1E_1^* & B_1E_2^* \\ cc & B_2B_2^* & B_2B_3^* & B_2E_1^* & B_2E_2^* \\ cc & cc & B_3B_3^* & B_3E_1^* & B_3E_2^* \\ cc & cc & cc & E_1E_1^* & E_1E_2^* \\ cc & cc & cc & cc & E_2E_2^* \end{bmatrix}$$

m = 0, 1, 2
for F0, F1, F2

Time Averaged Spectral Matrix (ASM)

$$\mathbf{ASM}\left(\omega_j^{(m)}\right) = \frac{1}{N_{SM}^{(m)}} \sum_{k=1}^{N_{SM}^{(m)}} \mathbf{SM}_k\left(\omega_j^{(m)}\right) = \langle \mathbf{SM} \rangle_{time}$$

***Frequency* average ...**

$$\mathbf{S}\left(\omega_j^{(m)}\right) = \langle \mathbf{ASM} \rangle_{frequency}$$

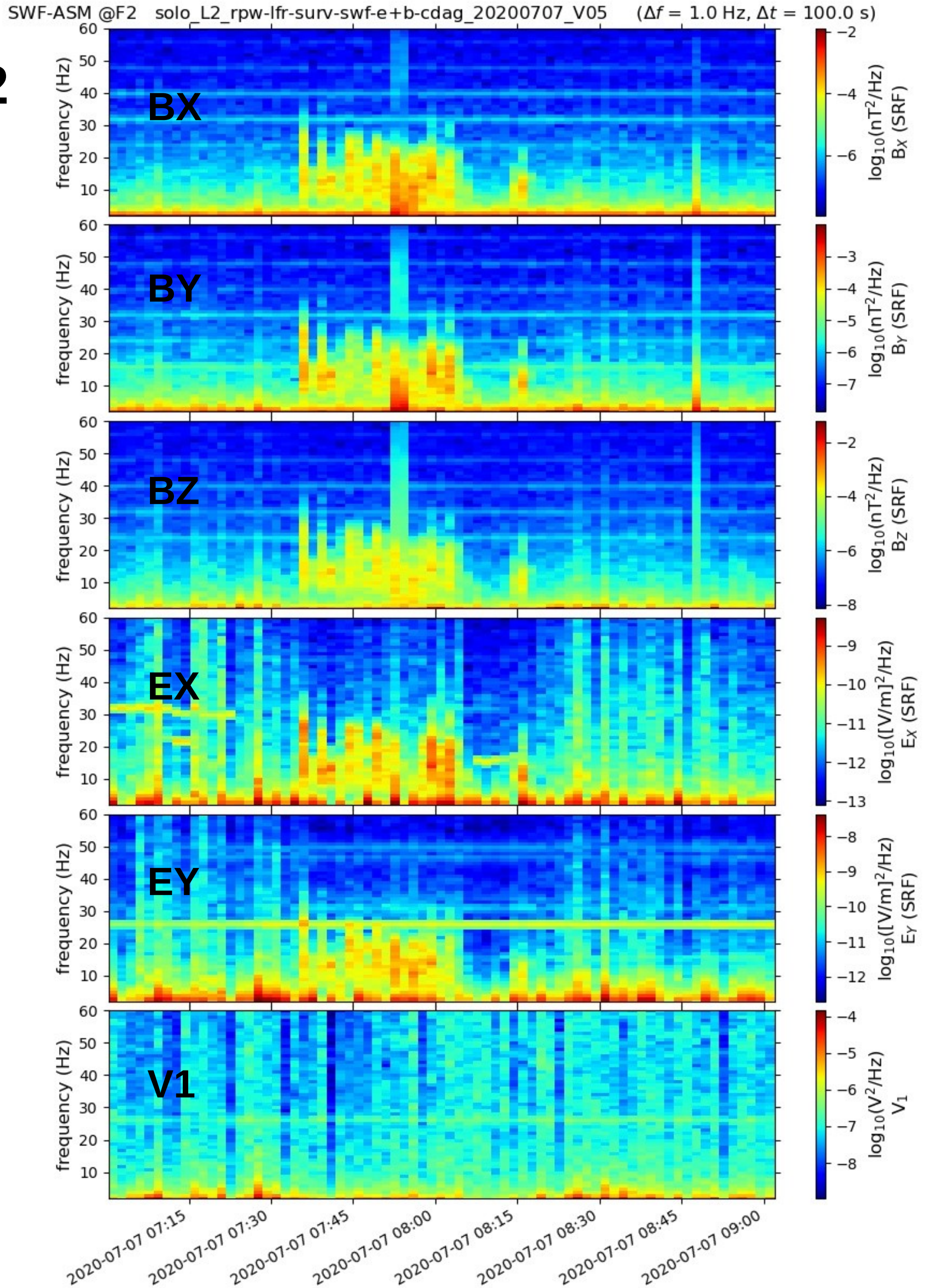
... before computations of the BPs (i.e. wave parameters)

	BP1 PB:	Power spectrum of the magnetic field (***B***)
	BP1 PE:	Power spectrum of the electric field (***E***) **=> kcoef**
Mono-**k** assumption: (Means, JGR, 1972)	**BP1** NVEC:	Wave normal vector (from ***B***)
	BP1 ELLIP:	Wave ellipticity estimator (from ***B***)
(Samson & Olson, GJRA, 1980)	**BP1** DOP:	Wave planarity estimator (from ***B***)
	BP1 SX:	X_{SRF}(radial)-component of the Poynting vector **=> kcoef**
$\mathbf{n} \times \mathbf{E} = \frac{\omega}{k}\mathbf{B}$ →	**BP1** VPHI:	Phase velocity estimator **=> kcoef (patch needed)**
	BP2 AUTO:	Autocorrelations
$\frac{S_{ij}}{\sqrt{S_{ii}S_{jj}}}$ →	**BP2** CROSS:	Normalized cross correlations

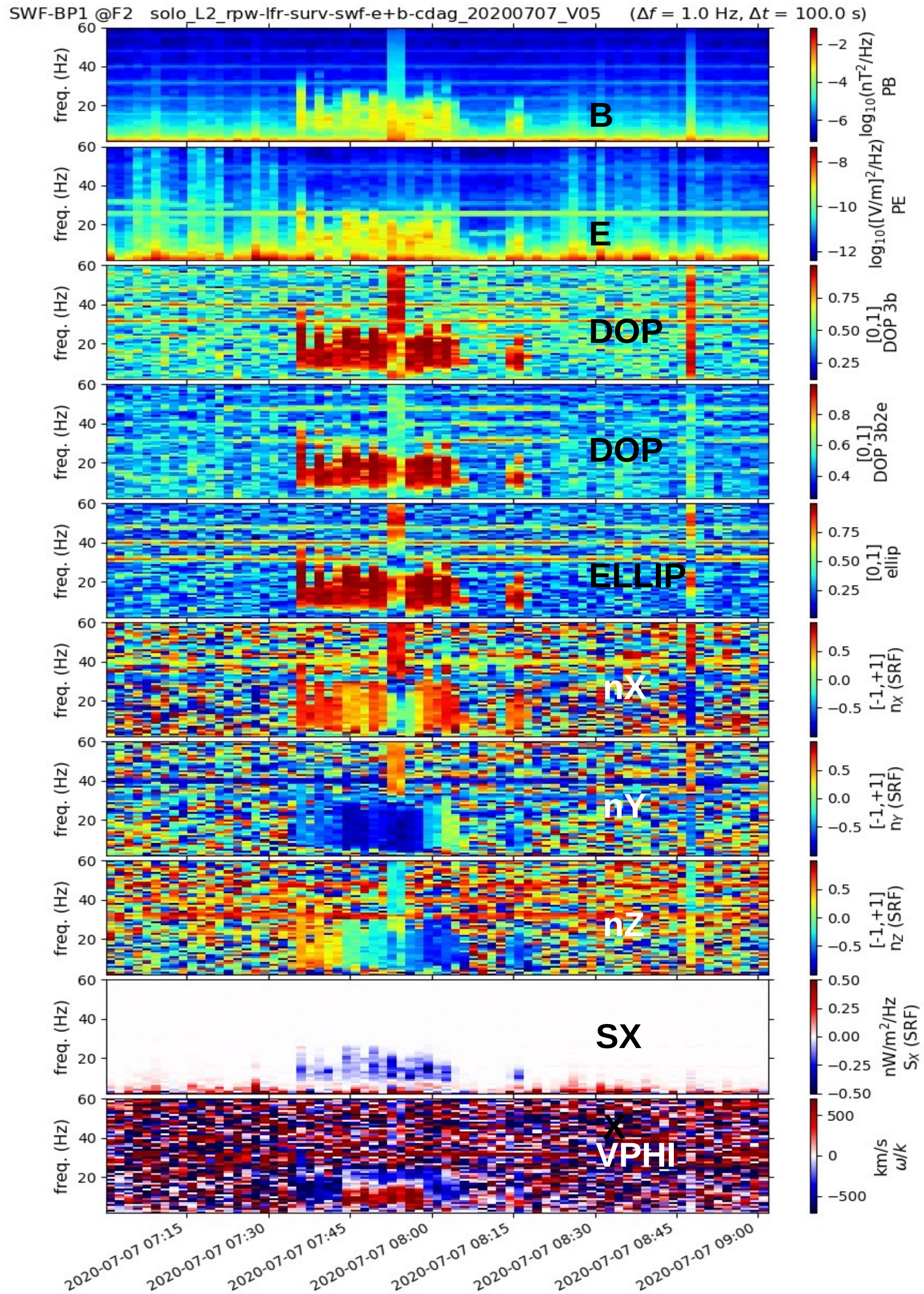
07/07 **2020**

ASM from SWF @F2

L2



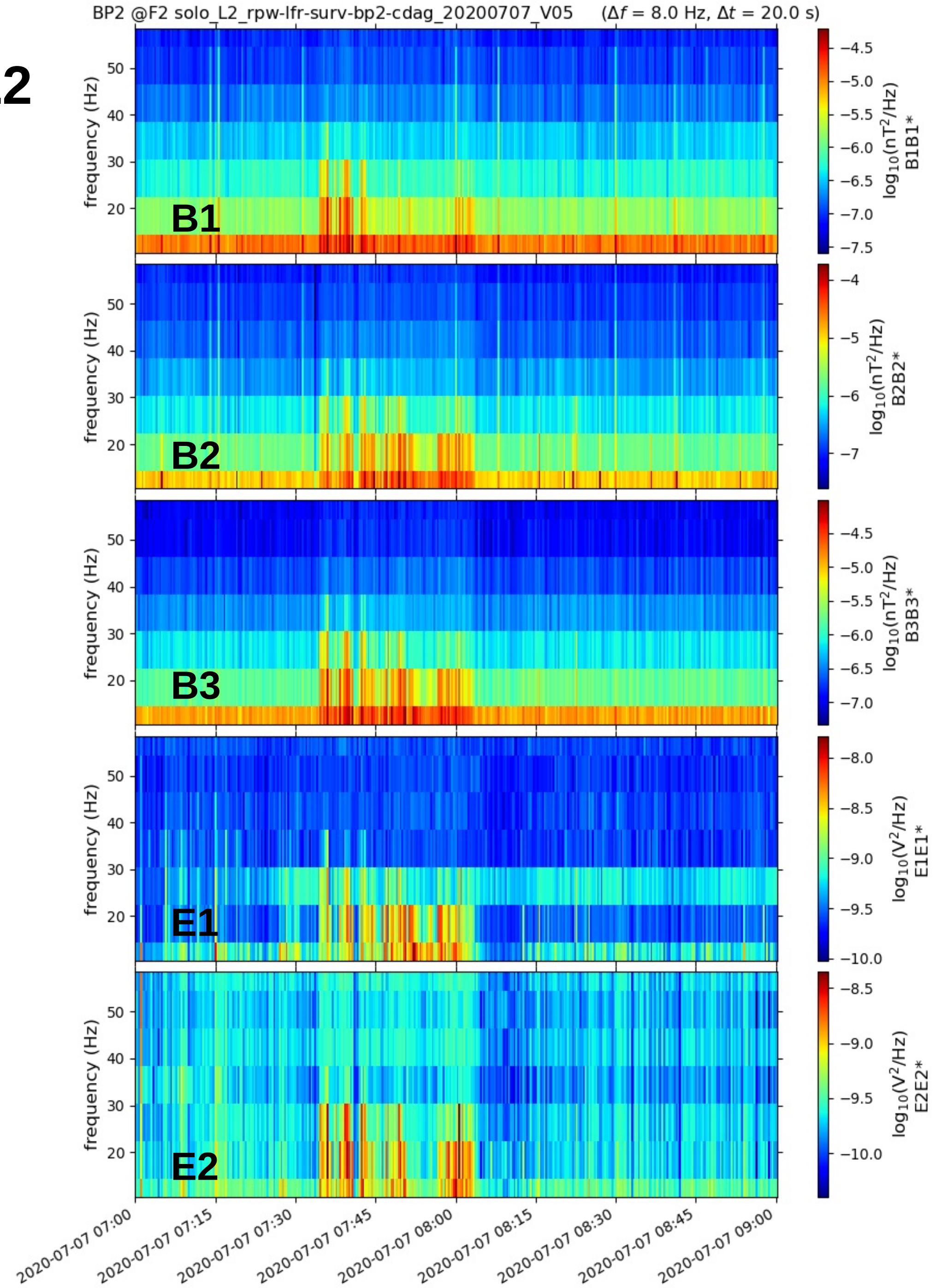
BP1 from SWF @F2



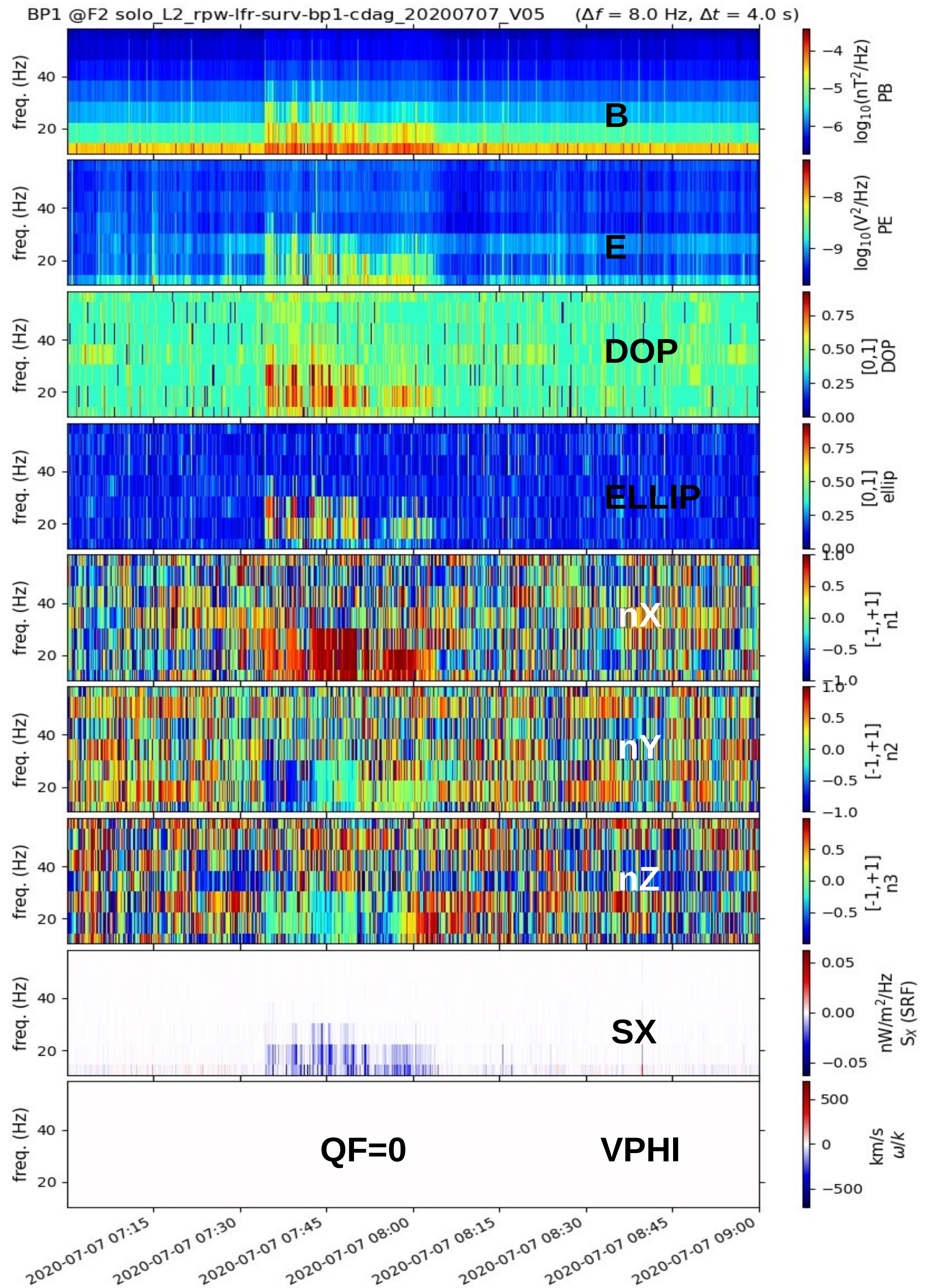
07/07 **2020**

BP2 @F2

L2



BP1 @F2

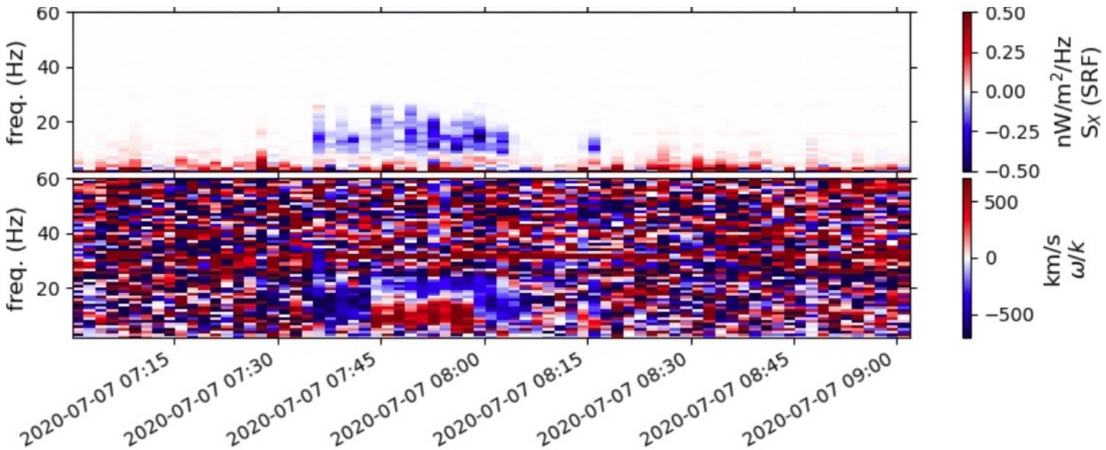


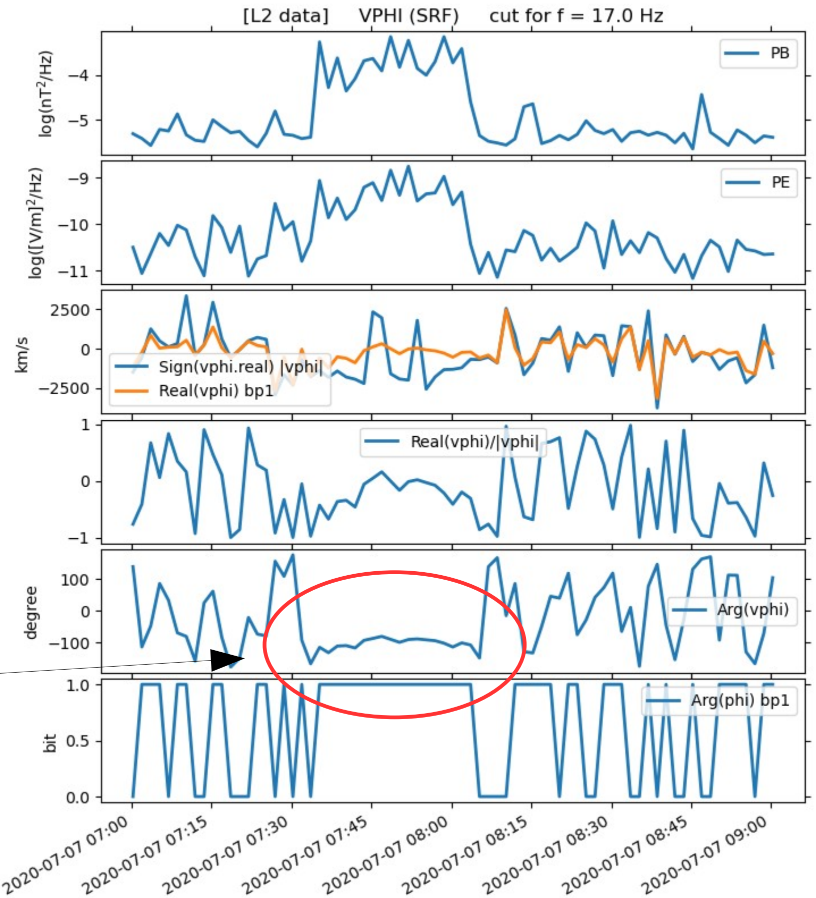
Computation of the phase velocity (VPHI)

$$\mathbf{n}\times\mathbf{E} = \frac{\omega}{k}\mathbf{B} \longrightarrow (\mathbf{n}\times\mathbf{E})\cdot\hat{\mathbf{e}}_{X'_{SO'}} B^*_{X'} = \frac{\omega}{k} B_{X'} B^*_{X'} = (n_{Y'}E_{Z'} - n_{Z'}E_{Y'})B^*_{X'}$$

$$\longrightarrow v_{\varphi} = \frac{\omega}{k} = \frac{n_{Y'}\langle E_{Z'}B^*_{X'}\rangle - n_{Z'}\langle E_{Y'}B^*_{X'}\rangle}{\langle B_{X'}B^*_{X'}\rangle}$$

Shall be a real number !



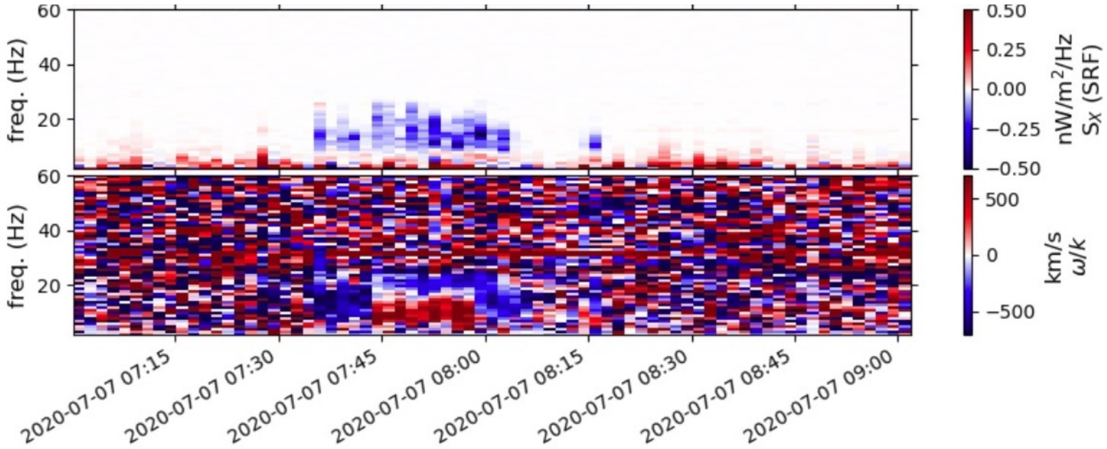


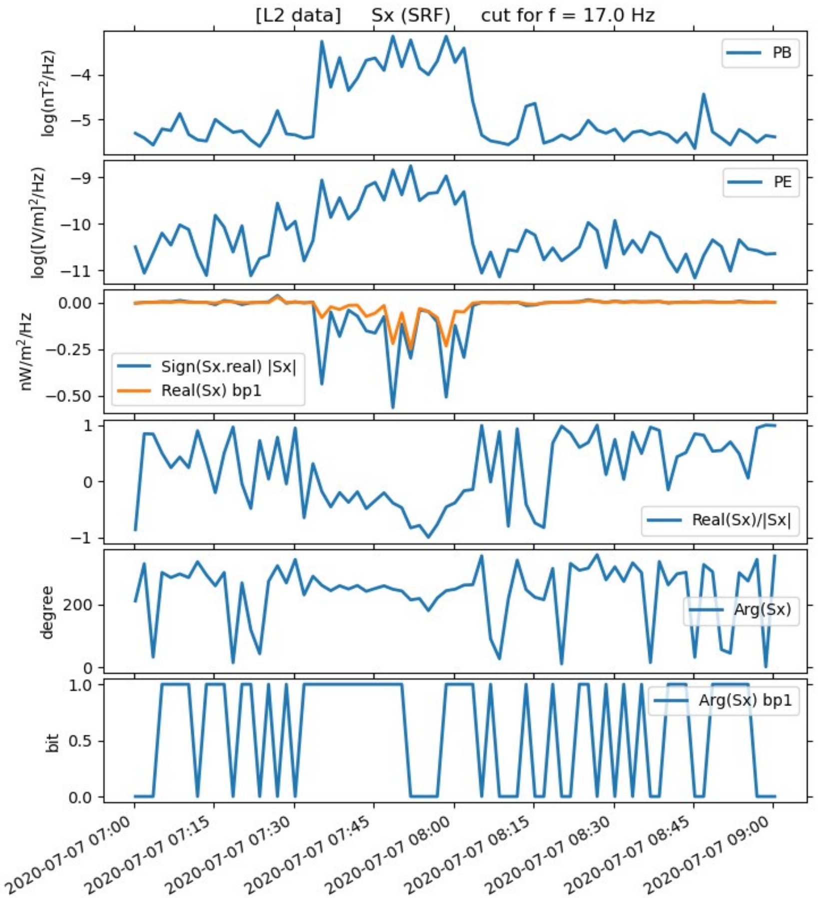
Indeed one obtains almost an imaginary number ! ?

Computation of the radial Poynting flux (SX)

$$\langle S_{X'} \rangle = \langle (\mathbf{E} \times \mathbf{B^*})_{X'} \rangle = \langle E_{Y'}, B^*_{Z'} \rangle - \langle E_{Z'}, B^*_{Y'} \rangle$$

Surprisingly far from a real number ?





Summary - Conclusion

- Ground segment software: no specific pb (Rodrigue Piberne)

- Last **update of the k-coefficients**: seems ok (analysis of KCOEFF_DUMP has also been started by Bruno Katra)

- **Phase shift issue** between electric (BIAS) and magnetic (SCM) data:
  - clear problem when computing the phase velocity,
  - questionable when computing the Poynting flux (SX)

Additional slides

Update of the LFR onboard k-coefficients (16 first / 32)

Has been done at the beginning of STP103 (06/07-12/07)

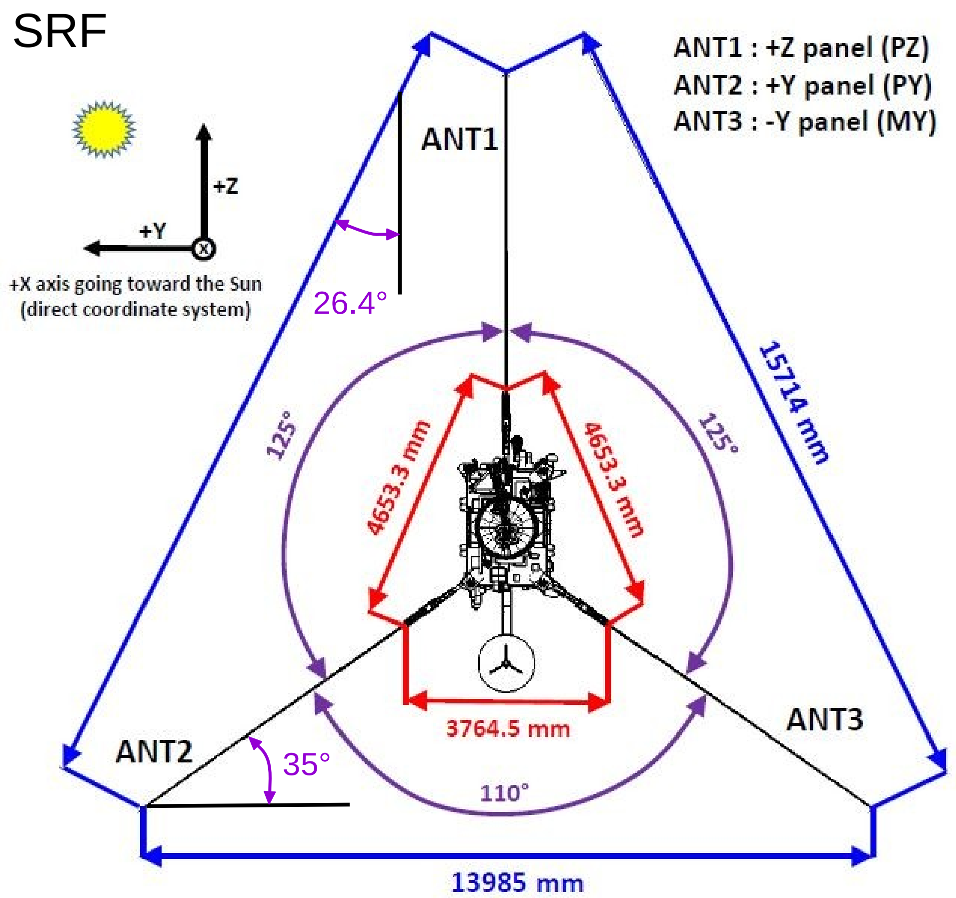
```
:      SOLO_CAL_RPW_BIAS_V202003101607.cdf
       SOLO_CAL_RPW-SCM_SCM-FS-MEB-PFM_V20200428000000.cdf
used:  SOLO_CAL_RCT-LFR-BIAS_V20190123171020.cdf
sed:   SOLO_CAL_RCT-LFR-SCM_V20190123171020.cdf
:      AC_DIFF_G5
       R = 0
       2020-06-18
```

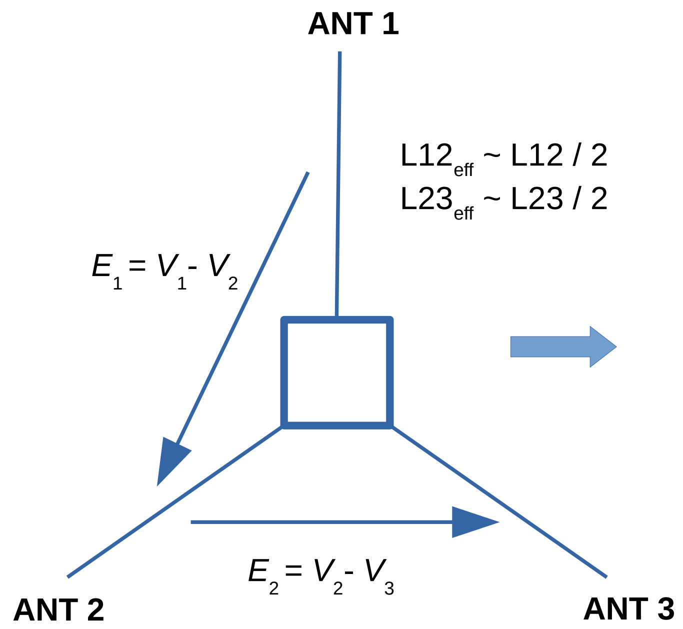
PE : transformation into SRF (2 ortho comp.)
SX : same for B + E-B relative calibration

requency (Hz)	kcoeff_1 (float)	kcoeff_2 (float)	kcoeff_3 (float)	kcoeff_4 (float)	kcoeff_5 (float)	kcoeff_6 (float)	kcoeff_7 (float)	kcoeff_8 (float)	kcoeff_9 (float)	kcoeff_10 (float)	kcoeff_11 (float)	kcoeff_12 (float)	kcoeff_13 (float)	kcoeff_14 (float)	kcoeff_15 (float)	kcoeff_16 (float)
1968.00	1.000000	1.250000	1.000000	-0.000000	0.680709	0.084467	-0.075378	0.002689	-0.006633	-0.004708	-0.828601	-0.100358	-0.615400	-0.121279	-0.684965	-0.072993
2736.00	1.000000	1.250000	1.000000	-0.000000	0.683845	0.046790	-0.077284	0.006480	-0.005590	-0.004755	-0.831959	-0.057230	-0.610072	-0.080905	-0.688697	-0.009801
3504.00	1.000000	1.250000	1.000000	-0.000000	0.684479	0.023171	-0.079053	0.009339	-0.004768	-0.004780	-0.831838	-0.030256	-0.601822	-0.055823	-0.691407	0.035243
4272.00	1.000000	1.250000	1.000000	-0.000000	0.684285	-0.000440	-0.080385	0.012018	-0.003927	-0.004456	-0.830962	-0.002683	-0.592844	-0.031602	-0.694529	0.078141
5040.00	1.000000	1.250000	1.000000	-0.000000	0.683102	-0.032862	-0.080902	0.015643	-0.003207	-0.004357	-0.828408	0.035232	-0.582548	0.000017	-0.694929	0.128425
5808.00	1.000000	1.250000	1.000000	-0.000000	0.679045	-0.077452	-0.080685	0.020638	-0.002670	-0.004105	-0.823017	0.088014	-0.569249	0.042608	-0.689826	0.190891
6576.00	1.000000	1.250000	1.000000	-0.000000	0.670473	-0.129371	-0.079861	0.026565	-0.002390	-0.003748	-0.812267	0.150402	-0.551233	0.090651	-0.677161	0.259483
7344.00	1.000000	1.250000	1.000000	-0.000000	0.660435	-0.171886	-0.078807	0.031303	-0.002101	-0.003724	-0.799841	0.200498	-0.530949	0.130834	-0.663375	0.317959
8112.00	1.000000	1.250000	1.000000	-0.000000	0.660779	-0.168973	-0.079255	0.030848	-0.001279	-0.003575	-0.798949	0.196290	-0.517772	0.135450	-0.669077	0.330162
8880.00	1.000000	1.250000	1.000000	-0.000000	0.677499	-0.077787	-0.082895	0.018871	-0.000068	-0.003632	-0.816823	0.084936	-0.519847	0.073541	-0.715673	0.249207
9648.00	1.000000	1.250000	1.000000	-0.000000	0.681470	0.022690	-0.084661	0.006578	0.000491	-0.003514	-0.819349	-0.036598	-0.515658	0.008594	-0.754707	0.152344
152.00	1.000000	1.250000	1.000000	-0.000000	0.250260	0.644123	-0.026544	-0.050484	-0.003991	-0.011837	-0.308949	-0.769155	-0.238878	-0.832557	-0.266696	-0.894962
280.00	1.000000	1.250000	1.000000	-0.000000	0.435058	0.536410	-0.045370	-0.038012	-0.004918	-0.008762	-0.530177	-0.636836	-0.422869	-0.688790	-0.463289	-0.733262
408.00	1.000000	1.250000	1.000000	-0.000000	0.533893	0.436249	-0.055709	-0.029291	-0.005497	-0.007015	-0.648099	-0.517161	-0.513439	-0.555338	-0.562708	-0.585619
536.00	1.000000	1.250000	1.000000	-0.000000	0.587681	0.359136	-0.061837	-0.022570	-0.006285	-0.006126	-0.713384	-0.424653	-0.558237	-0.453699	-0.611799	-0.472014
664.00	1.000000	1.250000	1.000000	-0.000000	0.618808	0.300719	-0.065342	-0.017659	-0.007364	-0.006185	-0.751394	-0.355742	-0.581250	-0.378724	-0.638215	-0.387033
792.00	1.000000	1.250000	1.000000	-0.000000	0.638057	0.256898	-0.067508	-0.013749	-0.007346	-0.005052	-0.774980	-0.303034	-0.594074	-0.323367	-0.652844	-0.323797
920.00	1.000000	1.250000	1.000000	-0.000000	0.650217	0.222509	-0.069381	-0.010659	-0.007270	-0.004845	-0.790133	-0.262800	-0.601775	-0.280832	-0.661824	-0.274364
1048.00	1.000000	1.250000	1.000000	-0.000000	0.659008	0.194906	-0.070656	-0.007736	-0.007396	-0.004562	-0.800838	-0.229373	-0.606966	-0.246803	-0.668286	-0.234006
1176.00	1.000000	1.250000	1.000000	-0.000000	0.665408	0.169230	-0.071854	-0.006071	-0.007568	-0.004507	-0.809481	-0.199492	-0.610956	-0.217468	-0.673320	-0.198444
1304.00	1.000000	1.250000	1.000000	-0.000000	0.670761	0.146243	-0.072662	-0.003866	-0.007289	-0.004516	-0.815947	-0.172497	-0.614190	-0.191600	-0.677752	-0.167006
1432.00	1.000000	1.250000	1.000000	-0.000000	0.674564	0.125896	-0.073488	-0.001698	-0.007126	-0.004639	-0.820815	-0.148492	-0.616342	-0.169058	-0.680765	-0.139056
1560.00	1.000000	1.250000	1.000000	-0.000000	0.677124	0.111511	-0.073890	-0.000368	-0.007175	-0.004652	-0.823828	-0.131910	-0.616715	-0.152930	-0.682387	-0.117971
1688.00	1.000000	1.250000	1.000000	-0.000000	0.677909	0.106444	-0.074376	0.000250	-0.006978	-0.004553	-0.824805	-0.125884	-0.615684	-0.145642	-0.682700	-0.106756
10.50	1.000000	1.250000	1.000000	-0.000000	-0.064163	0.691371	0.005238	-0.049219	0.001012	-0.010796	0.052509	-0.788477	0.071794	-0.856683	0.075569	-0.916724
18.50	1.000000	1.250000	1.000000	-0.000000	-0.138377	0.678801	0.008808	-0.050425	0.004210	-0.011572	0.136663	-0.788797	0.162978	-0.855399	0.174421	-0.916848
26.50	1.000000	1.250000	1.000000	-0.000000	-0.118979	0.682794	0.007025	-0.049553	0.001158	-0.010515	0.115928	-0.798322	0.146259	-0.866676	0.156269	-0.931405
34.50	1.000000	1.250000	1.000000	-0.000000	-0.086301	0.687445	0.004599	-0.050711	0.001742	-0.010776	0.079182	-0.809000	0.110229	-0.880463	0.116501	-0.944506
42.50	1.000000	1.250000	1.000000	-0.000000	-0.051160	0.691451	0.002795	-0.050467	0.002297	-0.008438	0.042965	-0.813893	0.074655	-0.888435	0.078498	-0.951999
50.50	1.000000	1.250000	1.000000	-0.000000	-0.017434	0.690637	-0.003165	-0.057706	0.007160	-0.013982	0.009704	-0.816750	0.037321	-0.898358	0.038160	-0.959488
58.50	1.000000	1.250000	1.000000	-0.000000	0.008883	0.692317	-0.001998	-0.051320	0.002576	-0.010018	-0.026732	-0.817230	0.010392	-0.894188	0.008285	-0.959643
66.50	1.000000	1.250000	1.000000	-0.000000	0.033912	0.691814	-0.006235	-0.051648	0.000089	-0.011058	-0.056269	-0.820826	-0.015915	-0.893520	-0.019992	-0.958181
74.50	1.000000	1.250000	1.000000	-0.000000	0.058759	0.689708	-0.008406	-0.051508	-0.000637	-0.010220	-0.085366	-0.818306	-0.041164	-0.890868	-0.047973	-0.954232
82.50	1.000000	1.250000	1.000000	-0.000000	0.081055	0.688097	-0.011295	-0.049474	-0.000468	-0.010783	-0.111820	-0.815674	-0.063871	-0.888986	-0.072991	-0.953335
90.50	1.000000	1.250000	1.000000	-0.000000	0.103966	0.685133	-0.012776	-0.050548	-0.001181	-0.011071	-0.137918	-0.812970	-0.087822	-0.884208	-0.099221	-0.947926
98.50	1.000000	1.250000	1.000000	-0.000000	0.122452	0.682203	-0.015249	-0.049152	-0.001474	-0.010516	-0.159991	-0.808639	-0.107694	-0.880608	-0.120736	-0.945213

11 F0 + 13 F1 + 12 F2 = 36 frequency bins

Approximate effective transfer matrix of ANT





A1Y ~ +3.5 m

A1Z ~ -7.04 m

A2Y ~ -6.99 m

A2Z ~ 0.0

$$\begin{bmatrix} E_1(\omega) \\ E_2(\omega) \end{bmatrix} = \begin{bmatrix} 0 & A_{1Y}(\omega) & A_{1Z}(\omega) \\ 0 & A_{2Y}(\omega) & A_{2Z}(\omega) \end{bmatrix}_{SRF} \cdot \begin{bmatrix} E_X(\omega) \\ E_Y(\omega) \\ E_Z(\omega) \end{bmatrix}_{SRF}$$

$$\begin{bmatrix} E_Y(\omega) \\ E_Z(\omega) \end{bmatrix}_{SRF} = \underbrace{\frac{1}{A_{1Y}A_{2Z} - A_{1Z}A_{2Y}} \begin{bmatrix} A_{2Z}(\omega) & -A_{1Z}(\omega) \\ -A_{2Y}(\omega) & A_{1Y}(\omega) \end{bmatrix}_{SRF}} \cdot \begin{bmatrix} E_1(\omega) \\ E_2(\omega) \end{bmatrix}$$

- Frequency dependence up to 10kHz is an open issue

E_Y = V23 / A2Y
E_Z = (2*V12 + V23) / 2 / A1Z

$$\mathbf{M}_{ANT\ \mathrm{to}\ SRF} \simeq \begin{bmatrix} 0. & -0.143 \\ -0.142 & -0.071 \end{bmatrix} (m^{-1})$$

Computation of the k-coefficients for PE

Power spectrum of the electric field

$$\langle E_{Y'} E^*_{Y'} + E_{Z'} E^*_{Z'} \rangle = \left\langle {}^{\mathrm{T}}\mathbf{E}_{ANT} \cdot \frac{1}{|A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}|^2} \begin{bmatrix} |A_{2Y'}|^2 + |A_{2Z'}|^2 & -A^*_{1Y'} A_{2Y'} - A^*_{1Z'} A_{2Z'} \\ -A_{1Y'} A^*_{2Y'} - A_{1Z'} A^*_{2Z'} & |A_{1Y'}|^2 + |A_{1Z'}|^2 \end{bmatrix} \cdot \mathbf{E}^*_{ANT} \right\rangle$$

$$= \frac{|A_{2Y'}|^2 + |A_{2Z'}|^2}{|A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}|^2} \left(S_{44} + \frac{|A_{1Y'}|^2 + |A_{1Z'}|^2}{|A_{2Y'}|^2 + |A_{2Z'}|^2} S_{55} - 2\,\Re \left[\frac{A^*_{1Y'} A_{2Y'} + A^*_{1Z'} A_{2Z'}}{|A_{2Y'}|^2 + |A_{2Z'}|^2} S_{45} \right] \right)$$

Calibration factor

$$\Rightarrow \quad \mathrm{PE} = S_{44}\, k^{pe}_{44} + S_{55}\, k^{pe}_{55} + \Re\left[S_{45}\, k^{pe}_{45} \right]$$

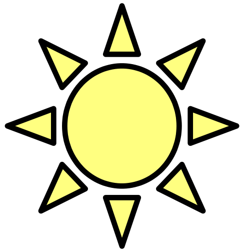
with

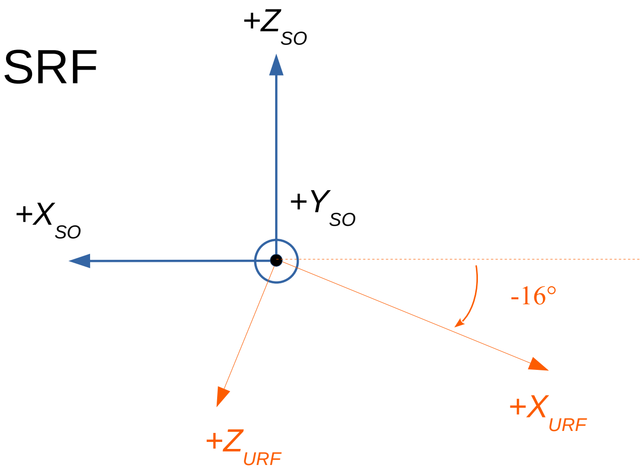
$$\begin{cases} k^{pe}_{44} = 1 \\ k^{pe}_{55} = \dfrac{|A_{1Y'}|^2 + |A_{1Z'}|^2}{|A_{2Y'}|^2 + |A_{2Z'}|^2} \\ k^{pe}_{45} = -2\, \dfrac{A^*_{1Y'} A_{2Y'} + A^*_{1Z'} A_{2Z'}}{|A_{2Y'}|^2 + |A_{2Z'}|^2} \end{cases}$$

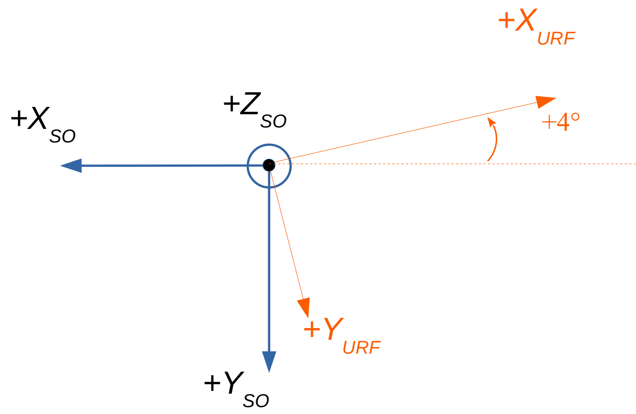
WARNING: The TF of BIAS and LFR are implicitly embodied in the TF matrix of ANT (just a common calibration factor)

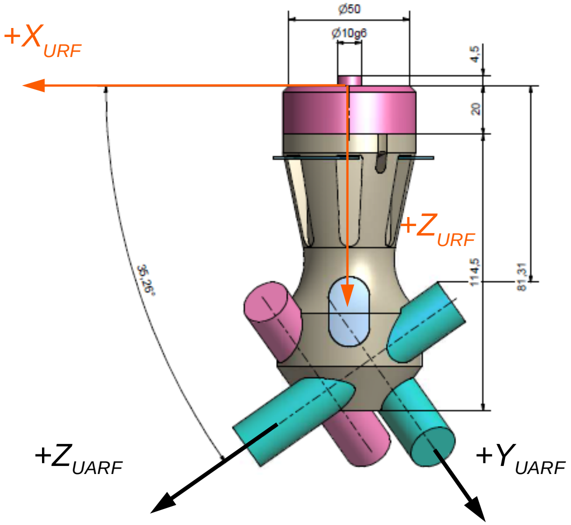
Current alignment of SCM

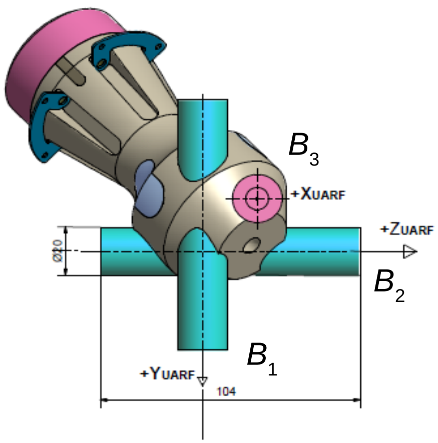
SRF











$$\overline{\overline{\mathbf{R}}}_{Y_{URF}}(+35.26^{\circ})\cdot \mathbf{Z}_{UARF} = \mathbf{X}_{URF}$$

$$\overline{\overline{\mathbf{R}}}_{X_{URF}}(+45^{\circ})\cdot \overline{\overline{\mathbf{R}}}_{Y_{URF}}(+35.26^{\circ})\cdot \mathbf{X}_{UARF} = \mathbf{Z}_{URF}$$

$$\overline{\overline{\mathbf{R}}}_{X_{URF}}(+45^{\circ})\cdot \overline{\overline{\mathbf{R}}}_{Y_{URF}}(+35.26^{\circ})\cdot \mathbf{Y}_{UARF} = -\mathbf{Y}_{URF}$$

Transformation matrices from UARF to SRF and from SCM to UARF coordinates:

$$\overline{\overline{\mathbf{M}}}_{SFR-UARF} = \begin{bmatrix} 0.501 & 0.600 & -0.624 \\ 0.744 & -0.667 & -0.0437 \\ -0.442 & -0.442 & -0.778 \end{bmatrix} \qquad \overline{\overline{\mathbf{M}}}_{UARF-SCM} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Transfer matrix of SCM and notations

$$\mathbf{B}_{SCM}(\omega) = \begin{bmatrix} B_1(\omega) \\ B_2(\omega) \\ B_3(\omega) \end{bmatrix}_{SCM} = \overline{\overline{\mathbf{A}}}_M(\omega)\cdot\mathbf{B}(\omega) = \begin{bmatrix} C_{1Y}(\omega) & C_{1Z}(\omega) & C_{1X}(\omega) \\ C_{2Y}(\omega) & C_{2Z}(\omega) & C_{2X}(\omega) \\ C_{3Y}(\omega) & C_{3Z}(\omega) & C_{3X}(\omega) \end{bmatrix}_{SCM} \cdot \begin{bmatrix} B_Y(\omega) \\ B_Z(\omega) \\ B_X(\omega) \end{bmatrix}_{SCM}$$

Normalized transfer matrix :

$$\overline{\overline{\mathbf{A}}}_M(\omega) = C_{1Y}(\omega) \times \overline{\overline{\mathbf{c}}}(\omega) = C_{1Y}(\omega) \begin{bmatrix} 1 & c_{1Z}(\omega) & c_{1X}(\omega) \\ c_{2Y}(\omega) & c_{2Z}(\omega) & c_{2X}(\omega) \\ c_{3Y}(\omega) & c_{3Z}(\omega) & c_{3X}(\omega) \end{bmatrix}_{SCM}$$

$$\Rightarrow \quad \begin{bmatrix} B_X(\omega) \\ B_Y(\omega) \\ B_Z(\omega) \end{bmatrix}_{SRF} = \overline{\overline{\mathbf{M}}}_{SRF-SCM} \cdot \left[\overline{\overline{\mathbf{A}}}_M^{-1}(\omega)\right]_{SCM} \cdot \mathbf{B}_{SCM}(\omega) = \frac{1}{C_{1Y}(\omega)} \underbrace{\overline{\overline{\mathbf{M}}}_{SRF-SCM} \cdot \left[\overline{\overline{\mathbf{c}}}^{-1}(\omega)\right]_{SCM}} \cdot \mathbf{B}_{SCM}(\omega)$$

$$\overline{\overline{\mathbf{M}}}_{SRF-SCM} = \overline{\overline{\mathbf{M}}}_{SRF-UARF} \cdot \overline{\overline{\mathbf{M}}}_{UARF-SCM}$$

$$\widetilde{\overline{\overline{\mathbf{M}}}}_{SRF} = \overline{\overline{\mathbf{M}}}_{SRF-SCM} \cdot \left[\overline{\overline{\mathbf{c}}}^{-1}(\omega)\right]_{SCM} = \begin{bmatrix} \widetilde{m}_{X1} & \widetilde{m}_{X2} & \widetilde{m}_{X3} \\ \widetilde{m}_{Y1} & \widetilde{m}_{Y2} & \widetilde{m}_{Y3} \\ \widetilde{m}_{Z1} & \widetilde{m}_{Z2} & \widetilde{m}_{Z3} \end{bmatrix}(\omega)$$

Computation of the k-coefficients for SX

X_{SRF}-component of the Poynting vector

$$\langle S_{X'} \rangle = \langle (\mathbf{E} \times \mathbf{B}^*)_{X'} \rangle = \langle E_{Y'} B^*_{Z'} \rangle - \langle E_{Z'} B^*_{Y'} \rangle$$

$$= \left\langle \frac{A_{2Z'} E_1 - A_{1Z'} E_2}{A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}} \frac{1}{C^*_{1Y}} \widetilde{m}^*_{Z'j} B^*_j \right\rangle - \left\langle \frac{-A_{2Y'} E_1 + A_{1Y'} E_2}{A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}} \frac{1}{C^*_{1Y}} \widetilde{m}^*_{Y'j} B^*_j \right\rangle$$

$$= \frac{\left(A_{2Y'} \widetilde{m}^*_{Y'j} + A_{2Z'} \widetilde{m}^*_{Z'j}\right) \langle E_1 B^*_j \rangle - \left(A_{1Y'} \widetilde{m}^*_{Y'j} + A_{1Z'} \widetilde{m}^*_{Z'j}\right) \langle E_2 B^*_j \rangle}{\left(A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}\right) C^*_{1Y}}$$

Calibration factor

$$= \frac{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}}{\left(A_{1Y'} A_{2Z'} - A_{1Z'} A_{2Y'}\right) C^*_{1Y}} \left[\frac{A_{2Y'} \widetilde{m}^*_{Y'j} + A_{2Z'} \widetilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}} S_{4j} - \frac{A_{1Y'} \widetilde{m}^*_{Y'j} + A_{1Z'} \widetilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}} S_{5j} \right]$$

$$\Rightarrow \quad \mathrm{SX'} = S_{41} k^{sx'}_{41} + S_{42} k^{sx'}_{42} + S_{43} k^{sx'}_{43} + S_{51} k^{sx'}_{51} + S_{52} k^{sx'}_{52} + S_{53} k^{sx'}_{53}$$

with

$$\begin{cases} k^{sx'}_{4j} = + \dfrac{A_{2Y'} \widetilde{m}^*_{Y'j} + A_{2Z'} \widetilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'} - A_{1Z'}A_{2Y'}}\right)\right] & j = 1, 2, 3 \\ k^{sx'}_{5j} = - \dfrac{A_{1Y'} \widetilde{m}^*_{Y'j} + A_{1Z'} \widetilde{m}^*_{Z'j}}{\sqrt{|A_{2Y'}|^2 + |A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'} - A_{1Z'}A_{2Y'}}\right)\right] \end{cases}$$

WARNING: As for ANT, the TF of LFR is implicitly embodied in the TF matrix of SCM (just a common calibration factor)

Computation of the k-coefficients for VPHI

$$\mathbf{n}\times\mathbf{E} = \frac{\omega}{k}\mathbf{B} \quad\longrightarrow\quad v_{\varphi} = \frac{\omega}{k} = \frac{n_{Y'}\langle E_{Z'}B^{*}_{X'}\rangle - n_{Z'}\langle E_{Y'}B^{*}_{X'}\rangle}{\langle B_{X'}B^{*}_{X'}\rangle}$$

Phase velocity

$$v_{\varphi} = \frac{C_{1Y}\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}}{A_{1Y'}A_{2Z'}-A_{1Z'}A_{2Y'}} \times \left[n_{Y'}\left(\frac{-A_{2Y'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} S_{4j} + \frac{A_{1Y'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} S_{5j} \right) - n_{Z'}\left(\frac{A_{2Z'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} S_{4j} - \frac{A_{1Z'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} S_{5j} \right) \right] /$$

$$\left[|\widetilde{m}_{X'1}|^2 S_{11} + |\widetilde{m}_{X'2}|^2 S_{22} + |\widetilde{m}_{X'3}|^2 S_{33} + 2\Re[\widetilde{m}_{X'1}\widetilde{m}^{*}_{X'2}S_{12}] + 2\Re[\widetilde{m}_{X'1}\widetilde{m}^{*}_{X'3}S_{13}] + 2\Re[\widetilde{m}_{X'2}\widetilde{m}^{*}_{X'3}S_{23}] \right]$$

Calibration factor

$$\begin{cases} \mathrm{VPHI} = \dfrac{\Re[\mathrm{NEBX'}]}{\mathrm{BX'BX'}} \\ \mathrm{ArgNEBX'} = \mathrm{Arg}[\mathrm{NEBX'}] \end{cases}$$

$$\begin{cases} \mathrm{NEBX'} = n_{Y'}(S_{41}k^{ny'}_{41} + S_{42}k^{ny'}_{42} + S_{43}k^{ny'}_{43} + S_{51}k^{ny'}_{51} + S_{52}k^{ny'}_{52} + S_{53}k^{ny'}_{53}) + \\ \qquad n_{Z'}(S_{41}k^{nz'}_{41} + S_{42}k^{nz'}_{42} + S_{43}k^{nz'}_{43} + S_{51}k^{nz'}_{51} + S_{52}k^{nz'}_{52} + S_{53}k^{nz'}_{53}) \\ n_{Y'j} = m_{Y'j}\,n_j \\ n_{Z'j} = m_{Z'j}\,n_j \qquad j = 1, 2, 3 \\ \mathrm{BX'BX'} = |\widetilde{m}_{X'1}|^2 S_{11} + |\widetilde{m}_{X'2}|^2 S_{22} + |\widetilde{m}_{X'3}|^2 S_{33} + 2\Re[\widetilde{m}_{X'1}\widetilde{m}^{*}_{X'2}S_{12}] + \\ \qquad 2\Re[\widetilde{m}_{X'1}\widetilde{m}^{*}_{X'3}S_{13}] + 2\Re[\widetilde{m}_{X'2}\widetilde{m}^{*}_{X'3}S_{23}] \end{cases}$$

$$\begin{cases} k^{ny'}_{4j} = \dfrac{-A_{2Y'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'}-A_{1Z'}A_{2Y'}}\right)\right] \\ k^{nz'}_{4j} = \dfrac{A_{2Z'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'}-A_{1Z'}A_{2Y'}}\right)\right] \\ k^{ny'}_{5j} = \dfrac{A_{1Y'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'}-A_{1Z'}A_{2Y'}}\right)\right] \\ k^{nz'}_{5j} = \dfrac{-A_{1Z'}\widetilde{m}^{*}_{X'j}}{\sqrt{|A_{2Y'}|^2+|A_{2Z'}|^2}} \times \exp\left[i\left(\varphi_{C_{1Y}} - \varphi_{A_{1Y'}A_{2Z'}-A_{1Z'}A_{2Y'}}\right)\right] \end{cases}$$